Serb. Astron. J. № 207 (2023), 1 - 7
DOI: https://doi.org/10.2298/SAJ2307001G

UDC 530.12 : 531.51
Original scientific paper

SOME APPLICATIONS OF THE SHAPIRO TIME DELAY

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(Received: October 4, 2023; Accepted: December 10, 2023)

SUMMARY: Listening to echoes has long been a way to estimate distances, a technique whose backbone is the time delay. The gravitational field also creates a time delay, called Shapiro time delay, that helps us extract some information from the field and is indeed due to the photon journey through the field. Here, the ability of the Shapiro effect to distinguish naked singularities from non-naked ones (black holes) is discussed. It is also inferred that this time delay may be hired to compare the various types of singularities with different dimensions. Besides them, the possibility of detecting the rotation of the assumed objects through surveying the gravitational time delay is also addressed.

Key words. Black hole physics – Methods: analytical – Gravitation

1. INTRODUCTION

Singularity is one of the attractive predictions of general relativity (GR) (Poisson 2004, Hawking and Penrose 2010). Although the Cosmic Censorship Hypothesis (CCH) generally rejects the existence of naked singularity (NS), there is not a common agreement on CCH, and actually, NS physics has a lot to say (Hawking and Penrose 2010). Consequently, NS formation has attracted plenty of attempts to itself (Janis et al. 1968, Yodzis et al. 1973, Shapiro and Teukolsky 1991, Joshi and Dwivedi 1993, Dwivedi and Joshi 1994, Christodoulou 1994, 1999, Joshi 2009, 2012, Crisford and Santos 2017, Zhang 2017). Subsequently, distinguishing NS from black holes (non-naked singularities, i.e. singularities quarantined from the surroundings by the event horizon) appears as a serious task for physicists, a task that is accomplished by studying various properties of NS (Virbhadra and Ellis 2002, Gyulchev and Yazadjiev 2008, Joshi et al. 2014, Liu et al. 2018, Shaikh and Joshi 2019, Ziaie et al. 2022, Zhdanov and Stashko 2020, Stashko and Zhdanov 2021, Stashko et al. 2021, 2023).

Indeed, recent advances in the field of photographing black hole candidates further encourage us to study NS and the differences with black holes. In this regard, observational properties of naked singularities have also been studied (Zhdanov and Stashko 2020, Stashko and Zhdanov 2021, Stashko et al. 2021, 2023). The Sagnac time delay also seems to be able to distinguish black holes from NS (Ziaie et al. 2022). In this setup, a satellite (as the sender/receiver) orbiting the object (NS or black hole) is crucial, a serious difficulty for the idea applicability, as such objects, are very far from us. Therefore, another type of time delay may be more useful for such studies, i.e. an experiment that can be done remotely (without any need to send a satellite to long distances).

The fourth test of general relativity, established by I. I. Shapiro (Shapiro 1964), is based on the time delay of light rays passing through the gravitational field. In addition to being used to check GR in the

Solar system (Shapiro et al. 1968, 1971, Biswas and Mani 2004), this effect is also helpful in verifying *i*) modified gravity theories (Asada 2008, Boran et al. 2018, Edelstein et al. 2021, Dyadina and Labazova 2022), generalized uncertainty principle (Ökcü and Aydiner 2021), *ii*) equivalence principle (Desai and Kahya 2018, Boran et al. 2019, Minazzoli et al. 2019, Kahya and Desai 2016), *iii*) the number of spacetime dimensions (Monteiro and Lemos 2017), and studying the Pulsars (Desai and Kahya 2018, Laguna and Wolszczan 1997, Desai and Kahya 2016, Pössel 2021, Ben-Salem and Hackmann 2022, Hackmann and Dhani 2019, Abbott et al. 2017). Fortunately, unlike the Sagnac effect, this experiment does not need to send a satellite and hence is possible remotely. Moreover, although picosecond has been reported as the accuracy order of Shapiro measurement (Will 2014), it seems that the next generation of gravitational detectors shall significantly increase the accuracy (Ballmer et al. 2010, Sullivan et al. 2020). The negative Shapiro time delay called the gravitational time advancement also happens when the light rays pass through a weaker gravitational field compared to the field at the observer's place, and it is predicted that the modern versions of the Michelson-Morley experiment shall measure this time advancement (Bhadra et al. 2023).

Indeed, although the Schwarzschild spacetime is the most general spherically symmetric vacuum solution and a simple solution, it is the backbone of our understanding of many phenomena like orbits and thermodynamics of black holes motivating physicists to study this metric and its various generalizations (Wiltshire et al. 2009, D'Inverno 1992, Chakrabarty and Tang 2023). Employing the Schwarzschild metric and some generalized forms of this spacetime including the Bardeen, Reissner-Nordström, and Ayón-Beato-García metric, it seems that the Shapiro time delay can distinguish these black holes from each other (Junior et al. 2023). On the other hand, to find constraints on the deviations from spherical symmetry (SS), focusing on the γ-metric (a non-spherically generalization of the Schwarzschild metric that, depending on the value of γ, can also present NS), the Shapiro time delay has been investigated (Chakrabarty and Tang 2023). The obtained time delay is equal to that of Schwarzschild meaning that the Shapiro time delay cannot be used to constrain γ, and hence, distinguish solutions with different γ (the criterion of deviation from SS). Therefore, it seems that it is not possible to distinguish NS from a black hole (even the simplest black hole solution i.e. the Schwarzschild black hole) by comparing the corresponding Shapiro time delays.

A long way has been traced to find the rotating version of the Schwarzschild geometry called Kerr metric which also plays a vital role in discovering the secrets of various phenomena (Wiltshire et al. 2009). Since the Schwarzschild metric is a vacuum solution, it is then obvious to look at all other black holes as its extensions. The Janis-Newman-Winicour (JNW) metric and its rotational version are two well-studied generalizations of the Schwarzschild and Kerr spacetimes, respectively (Janis et al. 1968, Gyulchev and Yazadjiev 2008). They can also include NS (Janis et al. 1968, Gyulchev and Yazadjiev 2008), and attract a lot of attention (see Refs. Janis et al. 1968, Virbhadra and Ellis 2002, Gyulchev and Yazadjiev 2008 and their citations). In five dimensions, the Myers-Perry (MP) metric is an extension of the Kerr black hole that reduces to the Kerr geometry whenever a 4-dimensional spacetime is taken into account. Correspondingly, the 4 and 5-dimensional Schwarzschild metrics are also recoverable if the zero limit of angular momentums is applied (Myers and Perry 1986). In summary, the background and importance of these geometries requires that they be studied.

Here, our first aim is to show the power of the Shapiro test in distinguishing black holes from NSs. To this end, the JNW metric (Janis et al. 1968) and its rotating version (Gyulchev and Yazadjiev 2008) are studied in the subsequent sections, respectively. The possibility of verifying rotation in the fifth dimension shall be studied in the fourth section by employing the MP metric (Myers and Perry 1986). A summary is also provided at the end.

2. SHAPIRO TIME DELAY IN JNW SPACETIME

In the presence of the scalar field $\Phi(=\frac{q}{2b\sqrt{\pi}}\ln(1-\frac{b}{r}))$ with charge q and mass M, the JNW metric is obtained as:

$$ds^2 = -H^\nu dt^2 + \frac{dr^2}{H^\nu} + H^{1-\nu}r^2(d\theta^2 + \sin^2(\theta)d\phi^2), \quad (1)$$

in which $H = 1 - \frac{b}{r}$, $\nu = \frac{2M}{b}$, and $b = 2\sqrt{M^2+q^2}$ (Janis et al. 1968, Virbhadra and Ellis 2002). The Schwarzschild spacetime is covered at the appropriate limit of $q = 0$ (or, $\nu = 1$). The spacetime represents a singularity at $r = b$ for $q \neq 0$ and has the photon sphere if $1/2 < \nu \leq 1$ (Janis et al. 1968, Virbhadra and Ellis 2002). Following the existence and absence of the photon sphere, a singularity is called strong NS if $0 \leq \nu \leq 1/2$, and it is called weak NS on the condition that $1/2 < \nu < 1$, respectively (Virbhadra and Ellis 2002). Indeed, it is easy to calculate the surface area at $r = b$ through:

$$A = \int_{\theta_0=0}^{\theta=\pi}\int_{\phi_0=0}^{\phi=2\pi}\left[H^{1-\nu}r^2\right]_{r=b} d\theta d\phi = 0, \quad (2)$$

for $0 \leq \nu < 1$ meaning that $r = b$ is an NS.

Fig. 1 displays a photon sent from point P towards E, while the path PE has the closest vertical distance μ from the object O (D'Inverno 1992) whose surroun-

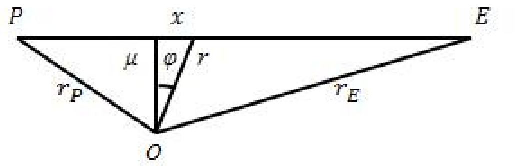

Fig. 1: Light ray moves towards E through the path PE.

ding spacetime is described by the JNW metric. In this manner, for a small displacement dx, we have:

$$\begin{cases} dr=\frac{x}{\sqrt{x^2+\mu^2}}dx \\ d\phi=\frac{\mu}{x^2+\mu^2}dx \end{cases} \Rightarrow d\phi = \frac{\mu}{r\sqrt{r^2-\mu^2}}dr. \quad (3)$$

Thus, since for a light ray ($ds^2 = 0$) on the $\theta = \frac{\pi}{2}$ plane

$$H^\nu dt^2 = \frac{dr^2}{H^\nu} + H^{1-\nu}r^2 d\phi^2, \quad (4)$$

one can easily use Eq. (3) to find:

$$dt^2 = H^{-2\nu}\big[1 + \frac{\mu^2}{r^2-\mu^2}H\big]dr^2, \quad (5)$$

as the time delay of a photon passing the path of PE in the presence of an object whose spacetime is described by the JNW metric. At long distances from O, one can approximate:

$$dt^2 \simeq \frac{r^2}{r^2-\mu^2}\big[1 + \frac{2b}{r}\nu - \frac{b\mu^2}{r^3} - \frac{2b^2\mu^2}{r^4}\nu\big]dr^2, \quad (6)$$

leading to:

$$dt \simeq \frac{rdr}{\sqrt{r^2-\mu^2}}\Big[1+\frac{2M}{r} - \frac{\mu^2}{r^2}(\frac{1}{r} + \frac{4M}{r^2})\sqrt{M^2+q^2}\Big], \quad (7)$$

where assuming $\frac{b}{r} \ll 1$, the Taylor expansion has been used (for example, we have $H^{-2\nu} \simeq 1 + \frac{2\nu b}{r}$) and terms including powers upper than $(\frac{1}{r})^4$ are ignored. In fact, unlike Ref. D'Inverno (1992), here, the $(\frac{1}{r})^4$ term is calculated and the Schwarzschild result is also obtainable by adopting $q = 0$ (or equally, $\nu = 1$) (D'Inverno 1992). Accordingly, the discrepancy with the Schwarzschild case emerges in terms including q. It is apparent that dt decreases as q increases meaning that, for the same dr, NS produces less dt compared to the Schwarzschild black hole, and accordingly, those NSs that have the photon sphere, generate more time delay rate compared to those without the photon sphere.

A Shapiro time delay measurement helps us confine and estimate the value of q. To show it, up to the first order of expansion, one can write Eq. (7) as $dt \sim dt_S - q^2\delta t$, where $dt_S \equiv dt\big|_{q=0}$ (the Schwarzschild case) and $\delta t \equiv \frac{\mu^2 dr}{2Mr^2\sqrt{r^2-\mu^2}}(1 + \frac{4M}{r})$. Now, consider an object with mass M and a measurement with uncertainty A reporting $t = \int dt$ for the time delay. If the object is supposed to be a Schwarzschild black hole, then mathematical calculations give us $t_S = \int dt_S$. In this manner, if $|t - t_S| < A$, then it can be said that with the accuracy $1 - A$, the object is a Schwarzschild black hole.

On the other hand, $|t - t_S| < A$ can also be used to find an upper bound on the value of q as $q^2 < \frac{A}{\int \delta t}$. Therefore, by increasing the precision of the setup, one can find a more accurate upper bound on q (or equally, one can find the value of q with more certainty). In this line, it is worthy to mention that although the current detectors have also significant precision i.e. $A \sim O(10^{-12})$s (Will 2014), the next generation of gravitational wave detectors equips us with more accurate measurements (Ballmer et al. 2010, Sullivan et al. 2020).

3. ROTATING 4-DIMENSIONAL METRIC

The geometry of spacetime including a rotating object (the rotational version of the JNW metric) is described as (Gyulchev and Yazadjiev 2008)

$$\begin{aligned} ds^2 = &-h^{1-\nu}\rho(\frac{dr^2}{\Delta} + d\theta^2 + \sin^2\theta d\phi^2) + \\ &h^\nu(dt - a\sin^2\theta d\phi)^2 + 2a\sin^2\theta(dt - a\sin^2\theta d\phi)d\phi, \end{aligned} \quad (8)$$

where:

$$\begin{aligned} h &= 1 - \frac{br}{\rho}, \\ \nu &= \frac{M}{\sqrt{M^2+q^2}} = \frac{2M}{b}, \\ \Delta &= r^2 + a^2 - br, \\ \rho &= r^2 + a^2\cos^2\theta, \end{aligned} \quad (9)$$

and the scalar field Φ takes the form:

$$\frac{q}{2b}\ln(1 - \frac{br}{\rho}). \quad (10)$$

In the above expressions, q and $a = J/M$ are the scalar charge and angular momentum per mass, respectively, while J denotes the angular momentum, and the JNW case is easily recovered at the $J = 0$ limit. It is reduced to the Kerr and Schwarzschild solutions for $q = 0 (\equiv \nu = 1)$ and $a = q = 0$, respectively. For other values of ν $(0 < \nu < 1)$, the line element Eq. (8) includes a naked singularity (Gyulchev and Yazadjiev 2008).

Now, considering the approach of the previous section, one reaches:

$$h^{\nu}dt^2+\frac{2\mu a}{r\sqrt{r^2-\mu^2}}(1-h^{\nu})drdt+ \tag{11}$$
$$[\frac{\mu^2a^2(h^{\nu}-2)}{r^2(r^2-\mu^2)}-(\frac{\mu^2}{r^2(r^2-\mu^2)}+\frac{1}{\Delta})h^{1-\nu}\rho]dr^2=0.$$

for the photon moving on the plane $\theta=\pi/2$, and easily, one can recover Eq. (5) at the appropriate limit $a=0$ leading to $h=H$. Of course, whenever $\theta=\pi/2$, we always have $h=H$ ($\rho=r^2$) and thus:

$$\begin{aligned}
&\frac{2\mu a}{r\sqrt{r^2-\mu^2}}(h^{-\nu}-1)\simeq\frac{4M\mu a}{r^2\sqrt{r^2-\mu^2}}\equiv-\Theta,\\
&\Delta\simeq r^2+a^2,\\
&(\frac{\mu^2h^{1-2\nu}\rho}{r^2(r^2-\mu^2)}+\frac{h^{1-2\nu}\rho}{\Delta})\simeq\\
&(1-\frac{b\mu^2}{r^3}-\frac{(r^2-\mu^2)a^2}{r^2(r^2+a^2)})\frac{r^2h^{-2\nu}}{r^2-\mu^2},\\
&\frac{\mu^2a^2(h^{\nu}-2)}{h^{\nu}r^2(r^2-\mu^2)}\simeq\\
&-\frac{\mu^2a^2}{r^2(r^2-\mu^2)}(1+\frac{2\nu b}{r})\simeq-\frac{\mu^2a^2}{r^2(r^2-\mu^2)}h^{-2\nu},
\end{aligned} \tag{12}$$

whenever $\frac{b}{r}\ll 1$. This finally gives the coefficient of dr^2 as:

$$\begin{aligned}
&-\frac{h^{-2\nu}r^2}{r^2-\mu^2}(1-\frac{b\mu^2}{r^3}-f(a))\simeq-\frac{r^2}{r^2-\mu^2}\Big[-\frac{b\mu^2}{r^3}\\
&(1-f(a))(1+\frac{2b}{r}\nu)-\frac{2b^2\mu^2}{r^4}\nu\Big]\equiv-\alpha,\\
&f(a)=\frac{\frac{a^2}{r^2}[1-\frac{\mu^2}{r^2}(2+\frac{a^2}{r^2})]}{1+\frac{a^2}{r^2}},
\end{aligned} \tag{13}$$

compared to Eq. (6) to see that, at this level of approximation, the effects of a are stored into $f(a)$. Therefore, at this limit, Eq. (11) takes the form $dt^2-\Theta drdt-\alpha dr^2\simeq 0$ (the alternative of Eq. (6)) that eventually renders:

$$\begin{aligned}
&dt\simeq\Big[\sqrt{\alpha}+\frac{\Theta(\frac{\Theta}{4\sqrt{\alpha}}+1)}{2}\Big]dr,\\
&\sqrt{\alpha}\simeq\frac{r}{\sqrt{r^2-\mu^2}}\Big[1-\frac{f(a)}{2}+(1-f(a))\frac{2M}{r}\\
&-\frac{\mu^2}{r^2}(\frac{1}{r}+\frac{4M}{r^2})\sqrt{M^2+q^2}\Big],
\end{aligned} \tag{14}$$

as the counterpart of Eq. (7) whenever $a\neq 0$ and of course provided that $f(a)\ll 1$. The plausibility of the latter condition is a reflection of our great distance from O in agreement with i) the primary assumption $b/r\ll 1$ and also, ii) the fact that such objects are very far from us. Indeed, it is the only solution of $dt^2-\Theta drdt-\alpha dr^2\simeq 0$ that produces Eq. (7) at the limit of $a=0$. Thus, the ability of the Shapiro time delay in detecting rotation in 4-dimensional spacetime as well as distinguishing a rotating NS from a rotating black hole is deduced.

4. THE MP SPACETIME

One of the Universe's mysteries is its number of dimensions which seems searchable using the Shapiro time delay (Monteiro and Lemos 2017). Another puzzle is the method of detecting motions in higher dimensions when we do not have direct access to the higher dimensions. Here, focusing on the MP metric, we are going to provide an answer by studying the effects of such a movement on the Shapiro time delay. The MP geometry is (Myers and Perry 1986)

$$\begin{aligned}
ds^2=&-dt^2+\frac{M(dt+a\sin^2\theta d\phi+\beta\cos^2\theta d\psi)^2}{r^2+a^2\cos^2\theta+\beta^2\sin^2\theta}\\
&+\frac{r^2(r^2+a^2\cos^2\theta+\beta^2\sin^2\theta)}{(r^2+a^2)(r^2+\beta^2)-Mr^2}dr^2\\
&+(r^2+a^2)\sin^2\theta d\phi^2+(r^2+\beta^2)\cos^2\theta d\psi^2\\
&+(r^2+a^2\cos^2\theta+\beta^2\sin^2\theta)d\theta^2,
\end{aligned} \tag{15}$$

where β is related to the angular momentum of fifth dimension.

For $\beta=a=0$, the five-dimensional Schwarzschild solution, i.e.

$$\begin{aligned}
ds^2=&-(1-\frac{M}{r^2})dt^2+\frac{dr^2}{1-\frac{M}{r^2}}\\
&+r^2(d\theta^2+\sin^2\theta d\phi^2+\cos^2\theta d\psi^2),
\end{aligned} \tag{16}$$

is achieved. Following the approach of the previous section, one finds:

$$dt^2=(1-\frac{M}{r^2})^{-2}\big[1+\frac{\mu^2}{r^2-\mu^2}(1-\frac{M}{r^2})\big]dr^2, \tag{17}$$

as the Shapiro time delay of a 5-dimensional Schwarzschild black hole whenever $\theta=\pi/2$ and $d\psi=0$. At long distances from O (or equally, $M/r\ll 1$), one easily reaches:

$$\begin{aligned}
&dt^2\simeq\frac{r^2}{r^2-\mu^2}[1+\frac{2M}{r^2}-\frac{M\mu^2}{r^4}]dr^2,\\
&\Rightarrow dt\simeq\frac{rdr}{\sqrt{r^2-\mu^2}}[1+\frac{M}{r^2}-\frac{M\mu^2}{2r^4}],
\end{aligned} \tag{18}$$

that clearly explains if the Shapiro time delay of a Schwarzschild candidate obeys this equation instead of the $q=0$ case of Eq. (7), then one can claim that the Universe has 5 dimensions. Compared to Eq. (7), it is seen that $1/r^3$ does not appear here, a fact that originated from the M/r^2 term in the metric Eq. (17) (for the Schwarzschild case, we have M/r). Moreover, it should be noted that the origin of r^{-4}

also differs from that of this term in Eq. (7) as their coefficients differ. Finally and as a preliminary check, if we only want to keep the first two terms $\big(1+\frac{M}{r^2}$ in this equation, and $1+\frac{2M}{r}$ in Eq. (7)$\big)$, then it is enough to replace M/r^2 and $2M/r$ with each other to see that the corresponding time delays are mutually recovered, a net reflection of the relationship between the corresponding metrics (Eqs. (5) and (17)).

A three-dimensional subspace of metric Eq. (15) with $\theta=\pi/2$ and $\psi=0$ gives the geometry:

$$\begin{aligned} ds^2 &= -(1-\frac{M}{r^2+\beta^2})dt^2+\frac{2Ma}{r^2+\beta^2}dtd\phi \\ &+\frac{r^2(r^2+\beta^2)}{(r^2+a^2)(r^2+\beta^2)-Mr^2}dr^2 \\ &+(r^2+a^2+\frac{Ma^2}{r^2+\beta^2})d\phi^2, \end{aligned} \tag{19}$$

leading to:

$$\begin{aligned} 0 &= -(1-\frac{M}{r^2+\beta^2})dt^2+\frac{2Ma\mu}{r(r^2+\beta^2)\sqrt{r^2-\mu^2}}dtdr \\ &+\big[(r^2+a^2+\frac{Ma^2}{r^2+\beta^2})\frac{\mu^2}{r^2(r^2-\mu^2)} \\ &+\frac{r^2}{r^2+a^2-\frac{Mr^2}{r^2+\beta^2}}\big]dr^2, \end{aligned} \tag{20}$$

when one uses Eq. (3) and considers a photon ($ds=0$). Clearly, Eq. (17) is produced for $a=\beta=0$, and additionally, even if the object M does not rotate in four dimensions ($a=0$), then the existence of the fifth dimension rotation still contributes to the results on the condition that $\beta\neq 0$. Hence, the detection of rotation in higher dimensions through the Shapiro effect is possible. When $a=0$ and $\beta\neq 0$, the solution is:

$$dt\simeq\frac{rdr}{\sqrt{r^2-\mu^2}}[1+\frac{M}{r^2+\beta^2}-\frac{M\mu^2}{2r^2(r^2+\beta^2)}], \tag{21}$$

where $\frac{M}{r^2+\beta^2}\ll 1$ has been assumed. Therefore, rotation in the fifth dimension is verifiable using this effect. Clearly, it also reduces to Eq. (18) when $\beta=0$. For the $\beta=0$ case (when $a\neq 0$), as the coefficient of $dtdr$ in Eq. (20) is not zero, by following the approach that led to Eq. (14), one finally obtains $dt^2-\varphi drdt-\epsilon dr^2\simeq 0$ yielding:

$$\begin{aligned} dt &\simeq \Big[\sqrt{\epsilon}+\frac{\varphi(1+\frac{\varphi}{4\sqrt{\epsilon}})}{2}\Big]dr, \\ \sqrt{\epsilon} &\simeq \frac{r}{\sqrt{r^2-\mu^2}}\Big[1-\frac{F(a)}{2}+(1-F(a))\frac{M}{r^2}-\frac{M\mu^2}{2r^4}\Big], \end{aligned} \tag{22}$$

as the only solution that recovers Eq. (18) at the limit $a=0$. Here, $\varphi=\frac{2Ma\mu}{r^3\sqrt{r^2-\mu^2}}$ and:

$$F(a)=\frac{a^2}{r^2}\Bigg(\frac{1-\frac{\mu^2}{r^2}\big[2+\frac{a^2}{r^2}-\frac{M}{r^2}\big]}{1+\frac{a^2}{r^2}-\frac{M}{r^2}}\Bigg), \tag{23}$$

and thus $\varphi=F(a)=0$ for $a=0$.

Generally, when $a,\beta\neq 0$, long calculations lead to $dt^2-\psi drdt-\mathcal{N}dr^2\simeq 0$ and thus:

$$dt\simeq\Big[\sqrt{\mathcal{N}}+\frac{\psi(1+\frac{\psi}{4\sqrt{\mathcal{N}}})}{2}\Big]dr, \tag{24}$$

where:

$$\begin{aligned} \sqrt{\mathcal{N}} &\simeq \frac{r}{\sqrt{r^2-\mu^2}}\Big[1-\frac{\mathcal{F}(a)}{2}+\frac{M(1-\mathcal{F}(a))}{r^2+\beta^2} \\ &- \frac{M\mu^2}{2r^2(r^2+\beta^2)}\Big], \\ \psi &= \frac{2Ma\mu}{r(r^2+\beta^2)\sqrt{r^2-\mu^2}}, \\ \mathcal{F}(a) &= \frac{a^2}{r^2}\Bigg(\frac{1-\frac{\mu^2}{r^2}\big[2+\frac{a^2}{r^2}-\frac{M}{r^2+\beta^2}\big]}{1+\frac{a^2}{r^2}-\frac{M}{r^2+\beta^2}}\Bigg). \end{aligned} \tag{25}$$

Clearly, Eqs. (21) and (22) are obtained at the appropriate limits $a=0$ and $\beta=0$, respectively. Of course, comparing Eqs. (21) and (18), it is understood that one could have achieved this result by replacing M/r^2 with $\frac{M}{r^2+\beta^2}$ in Eqs. (22) and (23).

In summary, while Eq. (21) implies on the implication of the rotation in the fifth dimension of the Shapiro time delay, difference between $F(a)$ and $f(a)$ clearly shows that even the existence of the fifth dimension affects the time delay ($\beta=0$). Indeed, as it is emphasized by the information stored in $\mathcal{F}(a)$, these effects become more tangible when $\beta\neq 0$. This achievement is strengthened by the point mentioned after Eq. (18), where the emergence of M/r^2 instead of M/r in the time delay of a five-dimensional Schwarzschild spacetime has been argued, a feature again confirmed by comparing Eqs. (22) and (14) with each other.

5. CONCLUSION

Depending on the metric, it is possible to distinguish NSs from black holes via the gravitational time delay (the Shapiro effect). Moreover, the possibility of verifying the existence of extra dimensions and detecting rotation in higher dimensions through the Shapiro time delay has also been studied. The results imply the ability of this effect in such investigations. Indeed, comparing Secs. (III) and (IV) shows that this effect can even be used to compare objects of various dimensions.

Therefore, we can hope that the use of the Shapiro time delay and such fundamental experiments shall help us analyze the dimensions of the universe and its contents. Finally, it is worthwhile to mention that challenging the ideas presented here by different data such as GW could be an interesting topic for future projects.

Acknowledgements – The authors would like to thank the anonymous referee for the valuable comments.

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НЕКЕ ПРИМЕНЕ ШАПИРОВОГ ВРЕМЕНСКОГ КАШЊЕЊА

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УДК 530.12 : 531.51
Оригинални научни рад

Радарско ослушкивање одјека се дуго користило за процену угаоног растојања, а у питању је метода која се заснива на временском кашњењу сигнала при проласку кроз гравитационо поље масивног тела, позната под називом Шапирово временско кашњење. Овај ефекат нам помаже да извучемо одређене информације о гравитационом пољу, а узрокован је путовањем фотона кроз ово поље. Овде се расправља о могућности да се Шапиров ефекат користи у разликовању нескривених сингуларитета од оних који су скривени (црне рупе). Закључено је да ово временско кашњење може да се искористи за упоређивање различитих врста сингуларитета и њихових различитих димензија. Осим тога, разматрана је могућност откривања ротације црних рупа на основу посматрања гравитационог временског кашњења.