Serb. Astron. J. № 191 (2015), 75 - 80
DOI: 10.2298/SAJ150915005S

UDC 524.6–34 + 524.6–323.8
Preliminary report

A SIMPLIFIED APPROACH TO KINEMATICS OF STARS IN SOLAR NEIGHBOURHOOD

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(Received: September 15, 2015; Accepted: October 19, 2015)

SUMMARY: For a number of stars from the solar neighbourhood the eccentricities of their galactocentric orbits are also calculated by using a simplified procedure given by Ninković (2011). Eccentricities are also calculated from the three-dimensional orbits around the centre of the Milky Way for the same sample of stars by applying a model with gravitational potential given analytically. The orbital eccentricities (radial span of each star) obtained in these two manners show a satisfactory agreement. It can be concluded that for typical thin-disk stars the use of such a simplified procedure which requires much less computing time per star to determine their motion around the Galactic centre, is justified.

Key words. Galaxy: kinematics and dynamics – solar neighbourhood

1. INTRODUCTION

The solar neighbourhood is known to be of exceptional importance for studying the structure and kinematics of the Milky Way. The reason is very simple, because almost all the necessary data are available for stars sufficiently close to the Sun. The quality of these data has grown substantially, especially after completing the Hipparcos mission. The Milky-Way kinematics in the solar neighbourhood or, more briefly, the local kinematics, had been known even before Hipparcos, for a variety of intriguing features (asymmetric drift, vertex deviation, star streams, etc). After Hipparcos, the local kinematics has been studied rather extensively (e.g. Dehnen and Binney 1998, Nordström et al. 2004, Alcobé and Cubarsi 2004, Cubarsi et al. 2010). It has become clear that the local stars are kinematically divided among thin disk, thick disk and halo (e.g. Bensby et al 2003). The local kinematics is also related to physical characteristics of stars (e.g. Aumer and Binney 2009). A complete explanation of all properties is, of course, impossible without stellar dynamics. In stellar dynamics there are two approaches based on equations of motion and on the Boltzmann equation between which there exists equivalence (in more detail, e.g. Cubarsi 2010b). The subject of the Boltzmann equation is the distribution of stars in the phase space which can also be represented through elements of their galactocentric orbits (e.g. eccentricities in Nordström et al. 2004). These are three-dimensional orbits, however. As a consequence, the definition of their elements is not always clear and, for this reason, the projections onto the meridional plane of orbits rather than well-defined orbital elements have often been presented in literature (e.g. Cubarsi 2010a). On the other hand, the notion of eccentricity, especially for a three-dimensional orbit, is also not quite clear (e.g. Ninković 2009). In addition, in order to determine the phase-space distribution we need to be able to calculate well-defined orbital elements for a large number of stars.

In the present paper this problem is solved by introducing two simply and clearly defined orbital elements, the mean distance and eccentricity, which can easily be determined for an arbitrarily large number of stars, but provided that we deal with stars of the thin disk.

2. THEORETICAL BACKGROUND

The motion of a star around, for instance, the centre of the Milky Way is studied by using the Lagrange equations:

$$\ddot{R} - R\dot{\theta}^2 = \frac{\partial \Pi}{\partial R} \ , \tag{1}$$

$$\frac{d}{dt}(R^2\dot{\theta}) = \frac{\partial \Pi}{\partial \theta} \ , \tag{2}$$

$$\ddot{Z} = \frac{\partial \Pi}{\partial Z} \ . \tag{3}$$

In these equations Π is the gravitational potential of the Galaxy, R, θ and Z are the cylindrical coordinates of the star with respect to the Milky-Way centre. If the potential is stationary and axially symmetric, then the two well-known integrals of motion are as follows:

$$E = \frac{1}{2}(\dot{R}^2 + \Theta^2 + \dot{Z}^2) - \Pi = \mathrm{const} \ , \tag{4}$$

$$J_z = R\Theta = \mathrm{const} \ . \tag{5}$$

Here $\Theta = R\dot{\theta}$, E is specific energy and J_z is the Z-component of specific angular momentum. The conservation of J_z is due to the axial symmetry and is obtainable from Eq. (2). As a consequence, Eq. (1) becomes

$$\ddot{R} - \frac{J_z^2}{R^3} = \frac{\partial \Pi}{\partial R} \ . \tag{6}$$

In the case of stellar disks the stars always remain sufficiently near the plane $Z = 0$ during their motion. Due to this circumstance in Eq. (6) the influence of the Z-motion becomes negligible. Then Eq. (6) may be regarded as a total differential equation. As well known, such an equation yields the following conservation

$$E_p = \frac{1}{2}(\dot{R}^2 + \Theta^2) - \Pi_p = \mathrm{const} \ , \tag{7}$$

where Π_p is the potential in the plane $Z = 0$, which, clearly, depends on R only. The conserved quantity Eq. (7) is a quasi integral because it is conserved only approximately (influence of $Z(t)$ is neglected in Eq. (6)). Since in Eq. (6) the potential variation is taken into account in the plane $Z = 0$ only, it is suitable to express its derivative in terms of the circular speed u_c

$$\ddot{R} - \frac{J_z^2}{R^3} = -\frac{u_\mathrm{c}^2}{R} \ . \tag{8}$$

The values of the two constants E_p and J_z determine the extrema of the distance R: the minimum R_p and maximum R_a. Instead of them one may use the following quantities:

$$R_\mathrm{m} = \frac{R_a + R_p}{2} \ , \tag{9}$$

$$e = \frac{R_a - R_p}{R_a + R_p} \ , \tag{10}$$

known as the mean distance (R_m) and eccentricity (e). If the eccentricity for a star orbit is low enough, then, as can easily be seen (Eqs. (9) and (10)), the interval $[R_p, R_a]$ will be sufficiently narrow so that it would be acceptable to assume, a power-law dependence:

$$u_\mathrm{c}(R) \propto R^\delta \tag{11}$$

for the circular speed within it. The procedure for obtaining the quantities defined by Eq. (9) and Eq. (10) by using Eq. (8) with assumption Eq. (11) is the same as it would be if the spherical symmetry were valid. In the case of spherical symmetry the quantities defined in Eq. (9) and Eq. (10) are exact integrals of motion because their constancy follows from the conservation of energy and angular momentum. Under the conditions of spherical symmetry, one can use the cumulative mass, which would also obey a power law analogous to Eq. (11). The relation between the exponents of the two power laws would be very simple. The corresponding formulae can be found in two papers by Ninković (1986, 2011).

If a sample of stars were given so that all of them are at the same position at given time t (say, $R = R_\odot$, $Z = 0$, same θ), then the values of orbit parameters Eq. (9) and Eq. (10) could be determined by application of the procedure described in the preceding paragraph, provided that the galactocentric velocity components $\dot{R}$ and Θ are known for all stars. This procedure is straightforward (in more detail described in Ninković 2011) but it is based on simplifying assumptions: the quasi integral of motion E_p Eq. (7) and the power law concerning the circular speed Eq. (11). Reliability of the so obtained results strongly depends on quality of these assumptions. For this reason, the motion of selected stars should be examined additionally. This can be done by assuming a model for the gravitational potential of the Milky Way in which the three-dimensional galactocentric orbit would be calculated by using Eq. (6) and Eq. (3) for each sample star. Such a procedure, though less straightforward, is more correct because it is free of simplifying assumptions. A comparison will indicate the quality of the simplified procedure.

3. MATERIAL AND METHOD

As said in the preceding section, the simplified procedure is applicable to objects moving in nearly planar orbits (Eq. (8)) of low eccentricity (Eq. (10)). When selecting stars, these conditions should be taken into account. Also, for clear reasons, the selected stars are expected to be from the solar neighbourhood.

The stars studied here are from the catalogue Spectroscopic Properties Of Cool Stars (SPOCS) by Valenti and Fisher (2005). This catalogue has been published and is available online by the VizieR catalogue service. It contains spectroscopic data for 1040 nearby F, G and K stars observed in the framework of the Keck, Lick and AAT planet search programs. High-quality echelle spectra were combined with V-band photometry and Hipparcos parallaxes to produce high-precision well-covered catalog of nearby stars. Out of these 1040 stars, for 1026 there exist all the data needed for the present analysis, in particular the star position, proper motion, radial velocity and distance (via trigonometric parallax).

The sample of 1026 stars has been already treated for the purpose of studying their galactocentric motion (e.g. Vidojević and Ninković 2009). According to these authors, a vast majority of the 1026 stars belongs to the thin disk. The motion of such stars around the Milky-Way centre is expected to occur along nearly planar orbits of low eccentricity.

Since the input data are given in the equatorial spherical reference system, it is necessary to transform them into the heliocentric Cartesian system in which the coordinate axes are along the galactic coordinates (longitude and latitude). For this purpose, the algorithm given in Johnson and Soderblom (1987) is applied. Furthermore, the obtained heliocentric Cartesian velocity components should be corrected for the solar motion, or motion of the Sun with respect to the Local Standard of Rest (LSR). As usually, the designations will be:

U direction towards Galactic centre: $l = 0°, b = 0°$;
V direction of Galactic rotation: $l = 90°, b = 0°$;
W direction towards the north Galactic pole: $b = 90°$.

Based on the most recent results (e.g. Dehnen and Binney 1998, Schönrich et al 2010, Bobylev and Bajkova 2010, Huang et al 2014) where a discrepancy is seen, in this paper the following values are assumed: $(U_\odot, V_\odot, W_\odot) = (10, 8, 7)$ for the velocity components of the solar motion; the unit is km s^{-1}.

The final star selection for the purpose of the present paper is done by using their velocities with respect to LSR. In particular, one bears in mind the result (Ninković et al., 2012) that more than 75% of all thin-disk stars from the solar neighbourhood are contained within a sphere centred on LSR with radius equal to 80 km s^{-1}. The selection is thus based on the space speed of a star with respect to LSR; only stars from the sample of 1026 stars for which the space speed with respect to LSR is less than (or equal to) 80 km s^{-1} are taken into account. Out of 1026 stars this condition is satisfied for 905 of them.

This approach in the final selection is further confirmed through the Toomre diagram for all 1026 stars (Fig. 1). A significant value along the ordinate can be due to a large $|W_{\rm LSR}|$ and then the simplified procedure can not be applied because the maximum distance from the Galactic plane can attain substantial values.

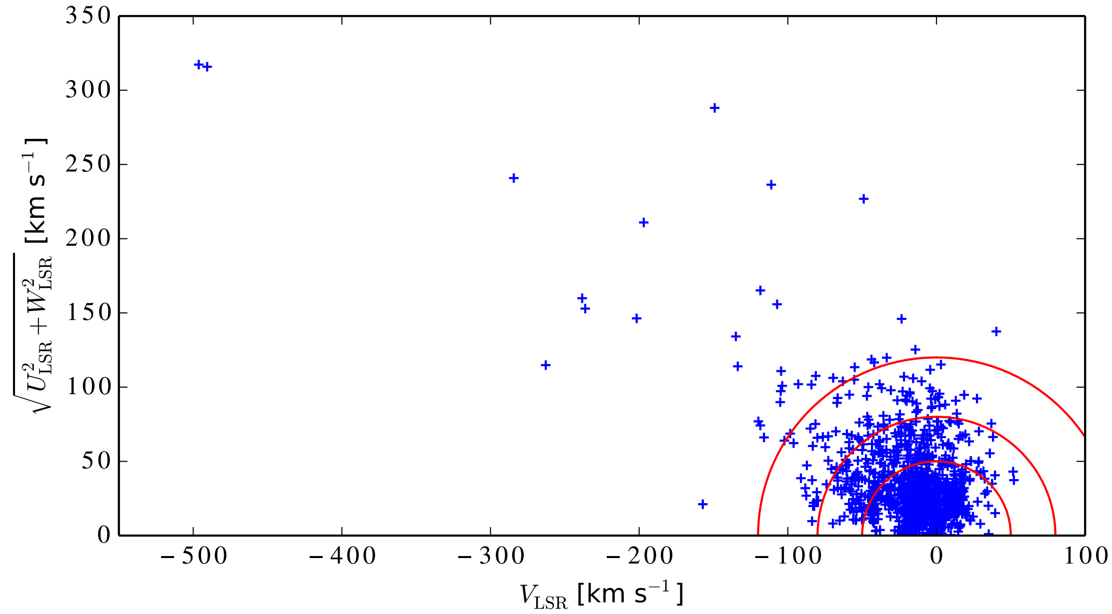

Fig. 1. *The Toomre diagram for the considered star sample. Lines indicate constant space speed* $v = \sqrt{U^2_{\rm LSR} + V^2_{\rm LSR} + W^2_{\rm LSR}}$, *for 50* km s^{-1}, *80* km s^{-1} *and 120* km s^{-1}.

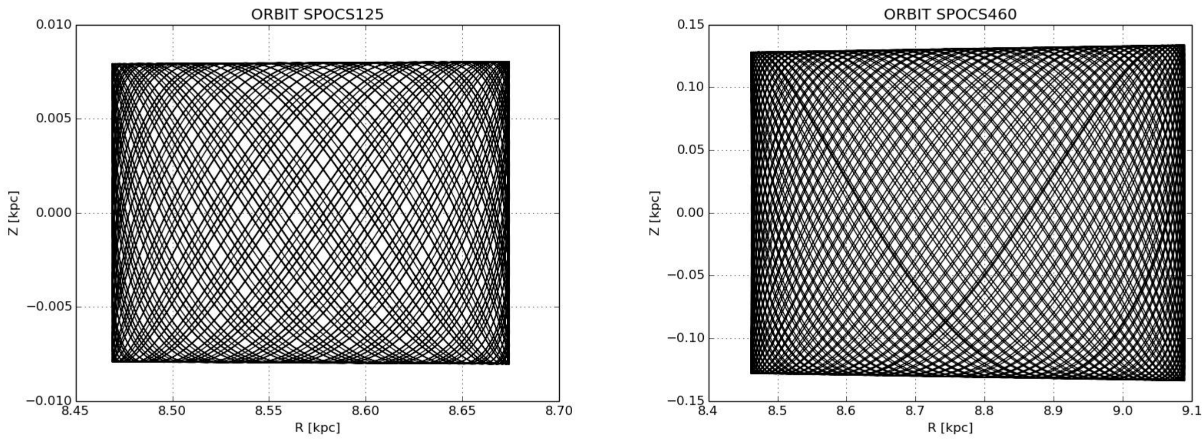

Fig. 2. *Meridional plot of calculated star orbits for SPOCS125 (left) and SPOCS460 (right). These orbits illustrate typical thin disk star candidates.*

The next step is to calculate galactocentric orbits in order to check the quality of the criterion for the selection of stars. We use the analytical expression for the Milky Way gravitational potential as given by the Ninković's (1992) model. The values for the distance of the Sun to the galactic rotation axis and the LSR speed are fixed in this model and equal to 8.5 kpc and 220 km s^{-1}, respectively. In the orbit calculation, it is taken into account that the Sun does not lie in the galactic plane; it is assumed that it is 0.015 kpc off northwards.

For the simplified procedure, the LSR speed and the exponent δ (expression Eq. (11)) are important. Clearly, for a meaningful comparison the values assumed for them must be equal to those from the model. The value of δ can be determined through the ratio α:

$$\alpha = \frac{A}{|B|} \ , \tag{12}$$

(A and B are the Oort constants), through the following relation:

$$\delta = \frac{1-\alpha}{1+\alpha} \tag{13}$$

as the interval within which the relation Eq. (11) is expected to be valid, contains $R_\odot$. In Ninković's (1992) model α is very close to unity. According to Eq. (13), δ is close to zero.

The galactocentric orbits are calculated by applying the Runge-Kutta 4th order method (Press et al. 2007) because of its great stability and precision. The motion duration is 10^{10} years. The energy (quantity E in Eq. (4)) is conserved up to ΔE/E $\sim 10^{-10}$.

4. RESULTS AND DISCUSSION

The obtained galactocentric orbits (example in Fig. 2) are characterized by the fact that the motion in R (R is the distance to the rotation axis) is practically unaffected. More precisely, the extremal values R_p and R_a are well defined and do not depend on $|Z|$ (distance to the galactic plane). These two values can be treated as orbital parameters. As an illustration of an orbit which is not nearly planar, the orbit for SPOCS 766 is given (Fig. 3). This star was rejected by the kinematical approach procedure we used here and also was classified as a typical halo star by Vidojević and Ninković (2009).

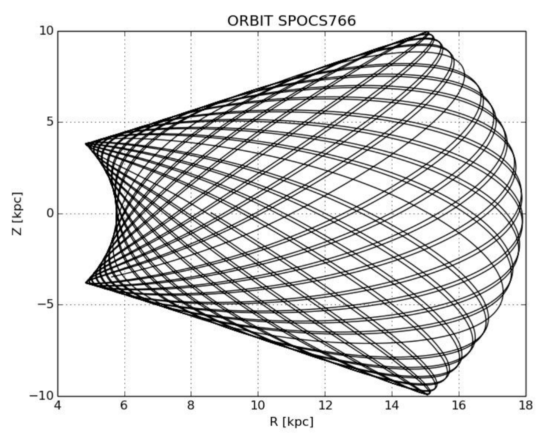

Fig. 3. *Meridional plot of the calculated star orbit for SPOCS766 showing star motion in the halo.*

To the subject treated here the motion perpendicularly to the galactic plane is not of importance. It is suitable to replace R_p and R_a by $R_{\rm m}$ and e, the mean distance and orbital eccentricity (Eqs. (9) and (10)).

Since the motion of thin-disk stars is studied, the eccentricity values are expected to be small. As a consequence, neither R_p nor R_a can be substantially different from the actual value of R. This means that the interval $[R_{p\,\min}, R_{a\,\max}]$ is expected to be rather narrow; $R_{p\,\min} \equiv \min\{R_p\}$ and $R_{a\,\max} \equiv \max\{R_a\}$ are the lowest and the highest values, respectively, within the sets R_p and R_a which concern all sample stars. In application of the Ninković (1992) model, the exponent δ is fixed, being determined by α. In Ninković's model α is very close to 1. In the course of the future work, we intend to apply the described simplified procedure to *varying* values of α, in order to compare the results for a large sample of stars with the measured circular velocities. This should be borne in mind if one applies a more simple procedure based on assumptions that the motion in R is practically unaffected by the perpendicular motion, and that a power-law dependence of the circular speed on R is acceptable.

In the simplified procedure, the values for $R_\odot$ and the LSR speed have no significant influence on results. The former quantity is not even among the input data. Though the latter quantity is necessary for obtaining the galactocentric velocity, it is further used as a unit so that only dimensionless quantities obtained after dividing by it are included in the calculations. In addition, the value for the distance of the Sun to the galactic plane is completely irrelevant. Therefore, the simplified procedure can easily be adapted to the procedure using three-dimensional galactocentric orbits in which values for $R_\odot$ and LSR are fixed, in accordance with the assumed model of the Milky Way. It is only important to assume the same values for the components of the velocity of the Sun with respect to LSR.

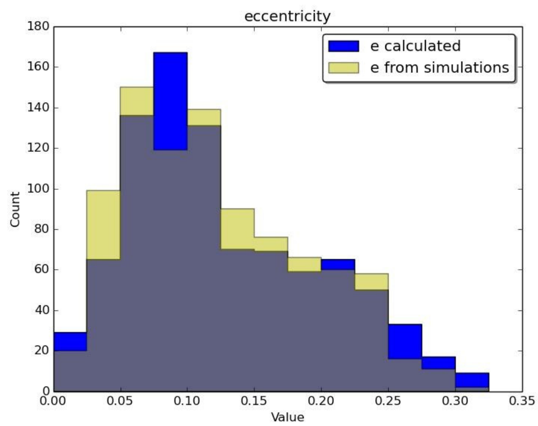

Fig. 4. *Eccentricity distribution for selected 905 stars: from simplified procedure (blue) vs. obtained from three-dimensional orbits (transparent yellow). Grey colour shows the region where results overlap.*

The distribution of the orbital eccentricities for selected stars (defined by Eq. (10)) is given in Fig. 4. As seen from Fig. 4., the eccentricity values are low, as expected for the thin disc, meaning that the mean distances should be close to the actual distance. For this reason the eccentricity distribution is more interesting than that of the mean distances. The eccentricity for each star is obtained twice, from the simplified procedure (described above), and also on the basis of the values for R_p and R_a that follow from the galactocentric orbits. The limits for R can be seen in panels in Fig. 2. As said above, the ratio α (Eq. (12)) is kept constant, equal to 1. The agreement of the results obtained by these two methods is satisfactory which proves that we can do fine tuning of the parameter δ to get the best value for the ratio of Oort constants in accordance with observations.

5. CONCLUSION

The present study shows that the applied simplified procedure can yield realistic results for orbital elements (eccentricity) of typical thin disk stars. In such a case, the computing time is significantly shortened. Also, the exponent for the circular speed (the ratio of Oort constants) can be varied. The idea is to continue the work by studying a much bigger sample. The velocity components of the solar motion could be also varied. In this way, the influence of these quantities would be studied in more detail with the possibility to obtain sufficiently reliable values for them. This is particularly important in light of the first results of the GAIA mission which are expected to be available soon.

Acknowledgements – During the work on this paper the author was financially supported by the Ministry of Education, Science and Technological development of the Republic of Serbia through the project ON176011, "Kinematics and dynamics of celestial bodies and systems".

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ПОЈЕДНОСТАВЉЕН ПРИСТУП КИНЕМАТИЦИ ЗВЕЗДА ИЗ СУНЧЕВЕ ОКОЛИНЕ

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УДК 524.6–34 + 524.6–323.8
Претходно саопштење

За један већи број звезда из Сунчеве околине рачунамо ексцентричности њихових галактоцентричних орбита користећи поједностављени поступак у коме је зависност кружне брзине од растојања степена функција. За те исте звезде рачунају се ексцентричности и рачунањем тродимензионе орбите око средишта Млечног пута применом модела у коме је потенцијал гравитације дат аналитички. Поређење добијених вредности ексцентричности (радијални распон за сваку звезду) показује задовољавајуће слагање. Може да се закључи да је код типичних звезда танког диска за одређивање њиховог кретања око средишта Млечног пута употреба овог поједностављеног поступка, за који је потребно много краће рачунарско време, оправдана.