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PR AND *PL* (PM_V) RELATIONS FOR CLASSICAL CEPHEIDS REVISITED

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SUMMARY: Using observational data available for a large number of Galactic Cepheids, we determine the relation between the radius and the period of pulsations, by means of a variant of the Baade–Wesselink method. Using, further, the brightness parameter according to the Barnes–Evans approach, we achieve our final goal, determination of the relation between the period and the mean luminosity. The coefficients in both relations are realistic. We indicate the need for accurate angular diameters of the Cepheids, which would be useful since the Cepheids are standard candles in the cosmic distance scale.

Key words. Cepheids – Stars: distances – Stars: statistics

1. INTRODUCTION

With the advent of new and different extragalactic distance indicators (Tip of the Red Giant Branch [TRGB], Tully-Fisher relations [TF], the Fundamental Plane and $D_n - \sigma$ [FP], Planetary Nebulae Luminosity Function [PNLF], Surface Brightness Fluctuations [SBF]), the question of the reliability of the primary distance indicator, classical Cepheids, becomes very important.

In the case of classical Cepheids accurate measurements of both, their physical parameters and distances, are very important because they offer the possibility to scale the Universe out to several megaparsecs thus forming the basis for evaluating the Hubble constant.

After Leavitt first noticed that bright Cepheids in the Small Magellanic Cloud [SMC] have longer periods than faint Cepheids, all observations indicated a Period–Luminosity (*PL*) relation of the form

$$< M_V >= a + b \log \mathcal{P}. \qquad (1)$$

Here $< M_V >$ is the mean absolute V magnitude, $\mathcal{P}$ is the period of absolute magnitude variation, whereas a and b are constants.

It is difficult to calibrate the intrinsic brightness of a Cepheid by standard techniques with an accuracy better than 10%. Some authors explain this by the low space density of Cepheids (e.g. Sasselov et al. 1990, 1992, 1994).

Direct trigonometric parallax measurements for even the five closest Galactic Cepheids do not appear feasible also in the near future (Monet et al. 1992), except perhaps for one of them (δ Cep) (see Gatewood et al. 1993). Most of the current work is therefore along a calibration path using the Baade-

Wesselink techniques (Baade 1926, Wesselink 1946, 1969, Welch et al. 1987, 1989).

The reason of the increasing interest in the Baade-Wesselink techniques lies in the development of the Barnes–Evans mathematical method (Barnes et al. 1976a, 1976b, 1977, 1978, 1988, Barnes 1980) and in resolving the disks of many Galactic Cepheids (e.g. Davis 1994). Precise interferometric measurements of the angular-diameter variation for a Cepheid, combined with its radial displacement computed from the integrated radial-velocity curve, allows a direct and very accurate distance determination. Applied to the Galactic Cepheids, this technique should eventually lead to a reliable zero point of the Cepheid distance scale (to 5% [0,1 mag] or better).

2. THE DATA AND METHOD USED

Our attempt in defining Cepheids as standard candles consists of the following steps:

(i) determination of the Period–Radius (PR) relation;

(ii) calibration of the surface brightness parameter F_V as function of the color V.

(iii) determination of the PL relation, in this case Period–Brightness (PM_V).

The first step is preceded by selecting a representative sample of Cepheids from astronomical databases. For this purpose we select 258 Galactic Cepheids listed in the General Catalogue of Variable Stars IV (GCVS IV) as the DCEP, DCEPS and CEP(B) types. Additional data for these stars, such as spectral type, spectral subtype and effective temperature, are taken from the SIMBAD database.

2.1. PR Relation

In this Section we explain how we determined the radius of a Cepheid variable and how we relate it to the period of pulsations (PR relation). To obtain a correct PR relation is very important because in the case of Cepheids in distant galaxies we have no meaningful radial velocity curves at our disposal, but it is still possible to obtain good light curves and their periods (Fouque–Gieren, 1997, Gieren 1982, 1989a,b, Gieren et al. 1989, 1990, 1993, Gieren and Brieva 1992).

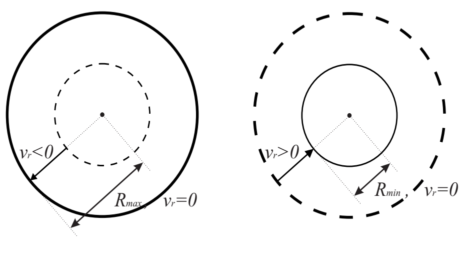

Fig. 1. *Changes of radial velocity during pulsations of a Cepheid.*

From the pulsating model of a variable star (see Fig. 1) we know that a positive radial velocity, $v_{\rm r} > 0$, indicates that the object is receding; if the sign is negative, $v_{\rm r} < 0$, then the object is approaching. The radius of variable star, $\mathcal{R}$, changes during the time and an indicator of this change is the radial velocity of the variable star. The mechanism of star pulsations could be expressed by means of the following scheme:

1. $t = t_0, v_{\rm r} = 0, \mathcal{R} = \mathcal{R}_{\rm min}$;
2. $t \in (t_0, t_1), v_{\rm r} < 0, \mathcal{R} \in (\mathcal{R}_{\rm min}, \mathcal{R}_{\rm max})$;
3. $t = t_1, v_{\rm r} = 0, \mathcal{R} = \mathcal{R}_{\rm max}$;
4. $t \in (t_1, t_2), v_{\rm r} > 0, \mathcal{R} \in (\mathcal{R}_{\rm max}, \mathcal{R}_{\rm max})$;
5. $t = t_2, v_{\rm r} = 0, \mathcal{R} = \mathcal{R}_{\rm min}$.

We approximate the radial velocity of a star, during the star pulsation period, $\mathcal{P}$, by using a function $\mathcal{V}_{\rm r} = f_{v_{\rm r}}(t),\ t \in [0, \mathcal{P}]$. The analytical form of the function $f_{v_{\rm r}}(t)$ is unknown. What we can be sure about is that the function, $f_{v_{\rm r}}$, should be equal to zero at two instants, that of the maximum of the star radius, $t_{\rm max}$, and of the minimum of the star radius, $t_{\rm min}$. Also, since the effective temperatures and spectral classes (with subclasses) are at our disposal, we can establish the mapping spectral class - effective temperature which enables us to obtain the values of the effective temperature at the epochs of zero radial velocity, denoted as $T_{\rm max}$ and $T_{\rm min}$.

The Baade-Wesselink method (e. g. Binney and Merrifield 1998, eq. (7.12), p. 400) offers the possibility to determine the ratio of the largest and smallest radius for a variable star when the ratios of the corresponding apparent magnitudes and effective temperatures are known. Since this relation is obtained from the luminosity formula, the apparent magnitude appearing there is the bolometric one. Thus, one should apply the bolometric correction.

In our case, the effective temperature range is from 5500K to 6500K and the corresponding bolometric correction is between -0.15 and -0.04. These values of the bolometric correction cannot affect significantly our results. As it is well known, the bolometric correction can have an important influence only for stars of spectral types O,B and M. For these reasons, the bolometric correction is here ignored.

The equation (from Binney and Merrifield 1998) mentioned above, rewritten for the present purpose, becomes

$$\eta = \frac{\mathcal{D}_{\rm max}}{\mathcal{D}_{\rm min}} = 10^{0.20(m_{\rm min}-m_{\rm max})-2.00\log\frac{T_{\rm max}}{T_{\rm min}}} \qquad (2)$$

As easily conjectured, η is the ratio of the extremal diameters of a Cepheid.

In order to estimate the mean radius of a Cepheid $< \mathcal{R} >$, we assume that the following relation (Swithart (1968))

$$\mathcal{V}_{\rm r}(t) = A_{v_{\rm r}} \sin\left(\frac{2\pi t}{\mathcal{P}}\right) \qquad (3)$$

is valid during the pulsation period. We calculate the amplitude A_{v_r} using LSQ . The equation of condition is:

$$A_{v_r} = \frac{\sum_{i=1}^{n} \sin\left(\frac{2\pi t_i}{\mathcal{P}}\right) \mathcal{V}_{r_i}}{\sum_{i=1}^{n} \sin^2\left(\frac{2\pi t_i}{\mathcal{P}}\right)}. \quad (4)$$

The integration of the expression for radius within the given boundaries, $\mathcal{R}_{\min}$ and $\mathcal{R}_{\max}$, leads to:

$$\int_{\mathcal{R}_{\min}}^{\mathcal{R}_{\max}} d\mathcal{R} = \mathcal{R}_{\max} - \mathcal{R}_{\min} = \\ = \int_0^{\mathcal{P}/2} \frac{\mathcal{P}A_{v_r}}{2\pi} \sin\left(\frac{2\pi t}{\mathcal{P}}\right) d\left(\frac{2\pi t}{\mathcal{P}}\right) = \frac{\mathcal{P}A_{v_r}}{\pi}. \quad (5)$$

$\mathcal{R}_{\max} = \eta \mathcal{R}_{\min}$ and we have $\mathcal{R}_{\min} = \frac{\mathcal{P}A_{v_r}}{\pi(\eta-1)}$.

It is well known that the radial velocity represents the change of radius over time:

$$\int_{\mathcal{R}_{\min}}^{\mathcal{R}} = \mathcal{R}(t) - \mathcal{R}_{\min} = \\ \int_0^t \frac{\mathcal{P}A_{v_r}}{2\pi} \sin\left(\frac{2\pi t}{\mathcal{P}}\right) d\left(\frac{2\pi t}{\mathcal{P}}\right) = \\ \frac{\mathcal{P}A_{v_r}}{2\pi}\left(1 - \cos\left(\frac{2\pi t}{\mathcal{P}}\right)\right). \quad (6)$$

Thus

$$\mathcal{R}(t) = \frac{\mathcal{P}A_{v_r}}{2\pi}\left(1 - \cos\left(\frac{2\pi t}{\mathcal{P}}\right)\right) + \frac{\mathcal{P}A_{v_r}}{\pi(\eta-1)}, \quad (7)$$

and the mean radius of a pulsating star is:

$$<\mathcal{R}>= \frac{1}{\mathcal{P}} \int_0^{\mathcal{P}} \mathcal{R}(t)\, dt = \\ = \frac{1}{\mathcal{P}} \int_0^{\mathcal{P}} \left(\frac{\mathcal{P}A_{v_r}}{2\pi}\left(1 - \cos\left(\frac{2\pi t}{\mathcal{P}}\right)\right) + \frac{\mathcal{P}A_{v_r}}{\pi(\eta-1)}\right) dt, \quad (8)$$

$$<\mathcal{R}>= \mathcal{P}A_{v_r} \frac{\eta+1}{2\pi(\eta-1)}. \quad (9)$$

Thus, one obtains the mean radius of a Cepheid as a function of the pulsation period, pulsation amplitude and ratio η. The amplitude and η can be found from equations (6) and (4), respectively. One of the constraints in our method is the assumption that the radial-velocity variation for the Cepheids has the sine form. The well-known property of the sine function to vanish at the beginning, middle and end of the period, results in eliminating those stars for which the rising times are close to one half of the pulsation period. Therefore, we use a total of 121 stars to obtain the following relation

$$\log <\mathcal{R}>= 0.557\,(\pm 0.181) \log \mathcal{P} \\ +0.964\,(\pm 0.167)\ , \quad (10)$$

where $\mathcal{P}$ is the period of pulsation, $< R >$ is in units of solar radius (see Fig. 2).

We can now compare our PR relation with that of Sachkov et al. (1998). Having only 62 Cepheids they obtained the following PR relation

$$\log <\mathcal{R}>= 0.620\,(\pm 0.030) \log \mathcal{P} \\ +1.230\,(\pm 0.030)\ . \quad (11)$$

The comparison is given in Fig. 2.

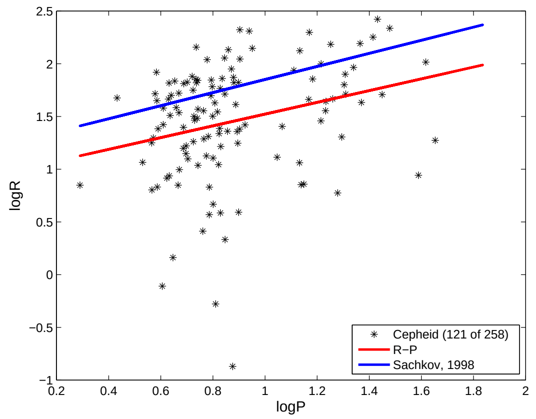

Fig. 2. *New relation compared with Sachkov et al. (1998) result.*

2.2. Relation surface brightness parameter F_V – color index V

The surface brightness parameter F_V, together with the visual magnitude, yields the angular diameter of a star through the relation of Barnes and Evans (1976),

$$F_V = 4.2207 - 0.1V_0 - 0.5 \log\theta \quad (12)$$

where V_0 is the unreddened apparent magnitude in the UBV system and θ is the angular diameter of the star in milliarcseconds. It can be shown that F_V is linearly related to the visual surface brightness S_V and can be calculated for stars of known angular diameter.

The surface brightness parameter is basically a temperature measure, and is based on a color index. As we know now, the best relationship is found for the index $(V-R)_0$ which is well defined for the entire range of stellar temperatures. There are no dependences on the luminosity class and on the interstellar extinction.

As we do not have enough stars with known angular diameters and R magnitudes in our sample, the general color conversion formula

$$R = V - 0.508\,(B-V) - 0.040\ , \quad (13)$$

is used for the purpose of calculating the color index $(V-R)_0$ i.e. unreddened apparent magnitude, V_0.

To calculate the angular diameters, we need the parallaxes; in our sample we find only 136 stars with positive Hipparcos parallaxes. An angular diameter is obtained from a 'right-angled triangle':

$$\theta = 2 \ \arcsin\left(\frac{1}{2}\frac{\mathcal{D}}{r}\right) \qquad (14)$$

where $\mathcal{D}$ is the linear diameter of the star and r is its heliocentric distance. We practically have $r >> \mathcal{D}$ and $\theta = \mathcal{D}/r$.

From (12), applying LSQ to the subsample of 136 Cepheid stars, we arrive to a new relation

$$F_V = 3.944(\pm 0.078) - 0.517(\pm 0.123)\,(V-R)_0\,, \quad (15)$$

where the value of the zero point is in a good agreement with the Barnes–Evans relation

$$F_V = 3.966(\pm 0.005) - 0.390(\pm 0.019)\,(V-R)_0\,, \quad (16)$$

but not the slope (see Fig. 3). An explanation for the difference in the slope, between our relation and the Barnes–Evans one, could be found in the non-homogeneous data sets. Namely, Barnes et al. used a different variable-star population (see Barnes et al. 1976, 1977).

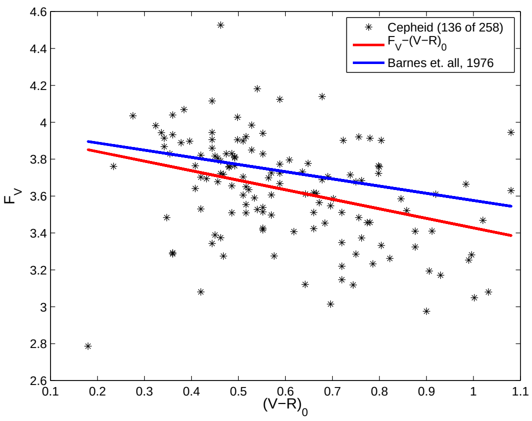

Fig. 3. *Our F_V versus Barnes-Evans relation.*

2.3. *PL* relation

The surface brightness parameter, F_V, can be related to the absolute V magnitude and radius of a Cepheid (Barnes et al. 1976). These authors recommend to use F_V instead of another quantity (surface brightness S_V) introduced by Wesselink (1969). So we have the following formula:

$$M_V - F_V + 5\log\mathcal{R} = 0. \qquad (17)$$

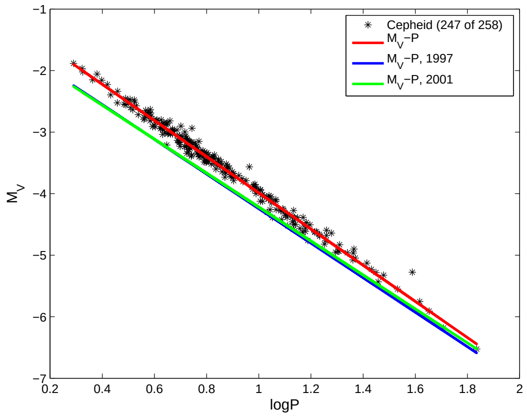

Fig. 4. *Our PL relation versus Feast and Catchpole (1997) and Freedman et al. (2001).*

Since we have already derived expressions for both $\log\mathcal{R}$ and F_v (Eq. 10 and Eq. 15), inserting them here and applying to the data concerning 247 out of 258 sample Cepheids we obtain the *PL* relation

$$M_V = -2.940\,(\pm 0.020)\log\mathcal{P} - 1.046\,(\pm 0.018) \quad (18)$$

In Fig. 4 we compare our relation with relations obtained by Feast and Catchpole (1997)

$$M_V = -2.81\log\mathcal{P} - (1.43 \pm 0.1). \qquad (19)$$

and Freedman et al. (2001)

$$M_V = -2.76\log\mathcal{P} - 1.458. \qquad (20)$$

3. DISCUSSION AND CONCLUSIONS

Our research is based on the elementary assumption accepted and exploited in many other recent articles. It pertains to the *PR* relation which appears to be linear up to the largest periods observed in the case of Cepheid variables. The data suggest that there is a universal *PR* relation obeyed by Cepheids.

The slope coefficient for the *PR* relation (eq 10) is close to 0.6 and there seems to be a reasonable agreement now concerning this value in a variety of empirical approaches and in the theoretical models. In this light, the significant difference of zero point of the *PL* relation existed.

Some authors suggest not to use the Hipparcos parallaxes as input data in deriving *PR* relation. Our analysis shows that no significant zero-point shift in the *PR* relation is produced due to the use of these parallaxes.

The disagreement in the constants when our *PL* relation is compared to some of widely accepted

PL formulae is mostly due to the uncertain estimation of relevant quantities and not well–known details of the theory of pulsation mechanism for the Cepheid variables. In the future work we will try to avoid the Euclidian geometry approach to the pulsation mechanism, and exploit possible differences in indexed ($_{\rm min}$) and ($_{\rm max}$) timing.

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ПОНОВНО ОДРЕЂИВАЊЕ *PR* И *PL* (PM_V) РЕЛАЦИЈА ЗА КЛАСИЧНЕ ЦЕФЕИДЕ

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Претходно саопштење

Користећи посматрачке податке доступне за велики број галактичких Цефеида одређујемо релацију између радијуса и периода пулсације. Осим варијанте Баде–Веселинкове методе користимо и параметар сјаја у складу са Барнс–Евансовим поступком и да би постигли главни циљ – одређивање релације између периода и средњег сјаја. Реалне вредности коефицијената у релацији указују на потребу прецизније оцене угаоних пречника цефеида. Тиме би се улога цефеида као стандардних свећа у дефинисању скале космичких даљина очувала.