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Preliminary report

A MODIFIED THEORETICAL $\Sigma - D$ RELATION FOR SUPERNOVA REMNANTS: I. THE CASE OF CONSTANT TEMPERATURE WITHIN THE SUPERNOVA REMNANT

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SUMMARY: We present a modification of the theoretical $\Sigma - D$ relation for supernova remnants (SNRs) in the adiabatic expansion phase. This modification is based on the convolution of the relation first derived by Shklovsky with the $\Sigma - D$ relation derived in this paper for thermal bremsstrahlung radiation from the ionized gas cloud. We adopt McKee & Ostriker's model for the components of the interstellar medium as part of our derivation. The modified Shklovsky theory agrees well with empirical results. Kesteven's modified theoretical relation gives the best agreement with the updated Galactic empirical $\Sigma - D$ relation.

Key words. ISM: supernova remnants – radiation mechanisms: thermal – radio continuum: ISM

1. INTRODUCTION

The relation between the surface brightness Σ and the diameter D of supernova remnants (SNRs) – the so-called $\Sigma - D$ relation – provides a convenient way to investigate the radio brightness evolution of SNRs. Shklovsky (1960a) presented a theoretical analysis of synchrotron radiation from an expanding spherical nebula, and the $\Sigma - D$ relation resulting from that analysis. This relation was also analyzed theoretically by Lequeux (1962), Poveda & Woltjer (1968), Kesteven (1968) and Duric & Seaquist (1986, hereafter D&S). Lequeux (1962) generalized the $\Sigma - D$ relation to the shell case to include the well-known shell-like SNR Cas A, and derived a relation which gave a better approximation than Shklovsky's relation by 10%. Poveda & Woltjer (1968) and Kesteven (1968) published two different supplements to the Shklovsky theory, but inconsistencies between observations and theory remained even afterwards. Poveda & Woltjer (1968) presented a model inspired by van der Laan theory (1962) where the magnetic field of the SNR remains constant as the SNR expands: the $\Sigma - D$ relation derived by those authors closely matched the empirical relation. Kesteven (1968) derived the relation for a shell-like SNR assuming that the thickness of the shell remained constant during the expansion. Finally, D&S derived a relation with the structure similar to that of Shklovsky: those authors adopted both Bell's version (1978a,b) of Fermi's accelerating mechanism and the magnetic field model described by Gull (1973) and Fedorenko (1983). Recent radio observations show that the surface brightness of SNRs decreases less rapidly than predicted by theory.

Radio observations of SNRs have confirmed the existence of the $\Sigma - D$ relation in the form predicted by the Shklovsky theory. Using the $\Sigma - D$ relation, Shklovsky (1960b) described a way to determine the distances to radio SNRs based on their surface brightness, assuming that this quantity does not depend on the distance to the radio SNR. The first empirical $\Sigma - D$ relation was derived by Poveda & Woltjer (1968). Milne (1970) derived an empirical $\Sigma - D$ relation and calculated distances to all 97 of the radio SNRs then known to exist in our galaxy. The relation itself was the subject of many investigations in order to determine precisely the distances to a specific set of calibrator sources and thereby improve the relation itself (e.g. Ilovaisky & Lequeux (1972), Sakhibov & Smirnov (1982), Huang & Thaddeus (1985)). Critical analyses of this relation have been conducted by Allakhverdiyev et al. (1983) and continued with the work of Green (1984) and Allakhverdiyev et al. (1986). Inaccurate calculations of the distances to certain calibrators are the main weakness of the relations derived in this manner: in other words, there are not enough SNRs with precisely calculated distances for the derivation of a proper $\Sigma - D$ relation (Green 1984). In addition, the ambient interstellar medium where supernovae have exploded must be taken into consideration when studying the relation. Allakhverdiyev et al. (1983, 1986) showed that the derivation of the $\Sigma - D$ relation is meaningful only for shell-like SNRs. During this period, Li & Wheeler (1984), Huang & Thaddeus (1985) and Berkhuijsen (1986) also considered the $\Sigma - D$ relation. From these first studies of this relation, significant differences between theoretical and empirical results were established. Interest and activity in the $\Sigma - D$ relation declined between the late 1980s and early 1990s, with Green (1991) showing that the calibrators are too scattered on the $\Sigma - D$ diagram to derive a valid relation. However, interest grew once again after Case & Bhattacharya (1998, hereafter C&B) derived an updated empirical $\Sigma - D$ relation – obtaining a much flatter slope than seen in earlier works – and determining distances for all known shell-like Galactic SNRs. Clearly, even after four decades of both theoretical and observational research, important aspects of the $\Sigma - D$ relation have not yet been completely explained.

The main topic of investigation in this paper is a theoretical $\Sigma - D$ relation for SNRs which takes into account thermal radiation from the interior of the SNR. The initial Shklovsky theory based on the synchrotron mechanism of radiation is supplemented with the bremsstrahlung radiation equations for the derivation of the modified $\Sigma - D$ relation. This derivation is mostly based on the McKee & Ostriker (1977, hereafter M&O) model for interstellar matter.

2. MODIFICATION OF THE THEORETICAL $\Sigma - D$ RELATION

Our understanding of the interstellar medium has continued to develop since the work of M&O: among other findings, we now know that supernova explosions have a tremendous influence on the distribution of interstellar matter. SNR shock waves dilute interstellar matter and increase its temperature, creating cavities of diffuse hot gas behind the expanding shock, with a typical density of $n = 10^{-2.5}$ cm^{-3} and temperature of $T = 10^{5.7}$K. When the shock wave sweeps up denser and colder interstellar clouds, the clouds evaporate and a hot environment is created. Consequently, the average shell SNR can stay in the adiabatic phase with diameters extending up to 360 pc. M&O describe the shell evolution in the early adiabatic phase and reveal that this phase lasts only until the SNRs reach a diameter of about 200 pc, with a second adiabatic phase following the first one for diameters greater than 200 pc. According to this model, very old SNRs (e.g. radio loops, such as OA 184) should be shell SNRs in the adiabatic phase, somewhere in between the early and later phases (see Urošević 2000, 2002, 2003). For an additional analysis of radio emission from SNRs, we will use the M&O model to describe a large diameter SNR with a hot interior and a denser and colder shell which forms after the adiabatic phase. Thermal radiation at radio wavelengths should be expected from this type of SNR. Also, thermal radiation from the diffuse medium is possibly described with bremsstrahlung radiation, that is, thermal radiation from the ionized gas cloud.

Shklovsky derived the $\Sigma - D$ relation of the form:

$$\Sigma = AD^{-\beta}, \tag{1}$$

directly from synchrotron radiation theory. In that theory, the thermal component of radiation is neglected even though it probably affects the $\Sigma - D$ relation. In this section it will be shown how the thermal component could influence the $\Sigma - D$ relation. Values for the exponent β in the empirical relations ($\beta = 2.38$) are less than values expected by theory ($\beta = 3.5$), and perhaps the empirical - theoretical inconsistency can be at least partially explained by the omission of the thermal component. If we derive the $\Sigma - D$ relation taking into account thermal radiation of the ionized gas cloud (that is, bremsstrahlung from the free electrons moving through the field of the positively charged ions) and in some way associate it with relations derived for the synchrotron mechanism, we may obtain a $\Sigma - D$ relation with a significantly reduced value for β.

The convolution method will be used to combine these two $\Sigma - D$ relations for different radiation mechanisms. Basically, this method describes how activity due to one process can influence another process, consequently producing a result that represents the combination of both processes.

2.1. $\Sigma - D$ relation for thermal radiation of the ionized gas cloud (case of constant temperature)

We will apply an algorithm used by Shklovsky for his synchrotron emission relation to the derivation of the $\Sigma - D$ relation for thermal radiation from the ionized gas interstellar cloud. Based on the theory of the bremsstrahlung radiation applied to an ionized gas cloud, we adopt a volume emissivity (e.g. Rohlfs & Wilson 1996) of the following form:

$$\varepsilon_\nu = \frac{4}{3}\frac{Z^2 e^6}{c^3}\frac{N_i N_e}{m^2}\Big(\frac{2m}{\pi kT}\Big)^{\frac{1}{2}} \ln\frac{p_2}{p_1}, \qquad (2)$$

where T is thermodynamic temperature of the medium, and N_i and N_e are the volume concentrations of the ions and electrons, respectively. The mass and charge of the electron are marked as m and e, while Z represents the atomic number. The collision parameter p represents the shortest distance between an ion and an electron in the course of the electron's accelerated motion in the ion field. The interval (p_1, p_2) includes all of the values that the collision parameter can take. The upper limit for p_2 should be the average distance between the ions (Debye length) while the limit values for parameter p_1 require quantum mechanical considerations. The previous equation is derived for interactions with large values of the collision parameter. Therefore, interactions among the particles are weak, considering the energy level. This is the reason why this theory is developed for straight-line motion of the electron in the ion field. The relation derived under these circumstances is very good for diffuse media where the particles are far away from each other and the energy change of the accelerated particles is small.

Assuming a constant value for the volume emissivity of the spherical ionized cloud, the radiation intensity can be written as:

$$I_\nu = \frac{1}{4\pi} R\varepsilon_\nu, \qquad (3)$$

where R is cloud radius. We assume that the temperature and concentration of particles do not change with changing distance from the center of the object (M&O model for the hot interstellar medium (HIM)). Therefore, the collision parameter is independent of the radius. According to Eqs. (2) and (3), we have:

$$\Sigma_\nu \propto R. \qquad (4)$$

From inspection of this relation, we notice that as the size of the SNR increases, its surface brightness also increases which is justified for the case of optically thin medium. Therefore, it is necessary to examine whether the medium is transparent for the specific frequency used for the construction of the $\Sigma - D$ relation (that is, 1 GHz).

The thermal radiation in the ionized gas cloud theory has the spectrum presented in Fig. 1. It clearly shows the frequency at which the optical depth τ is equal to unity. An expression used to determine this frequency where the spectrum is "breaking" has the following form (Rohlfs & Wilson 1996):

$$\nu_0 = 0.3045 T_e^{-0.643} (a(\nu, T) EM)^{0.476}, \qquad (5)$$

where ν_0 is expressed in GHz, T_e is the electron temperature in K, the correction factor $a(\nu, T)$ is approximately equal to 1 and the emission measure EM=$\int N_e^2 ds$ where N_e is in cm^{-3}, and s in pc.

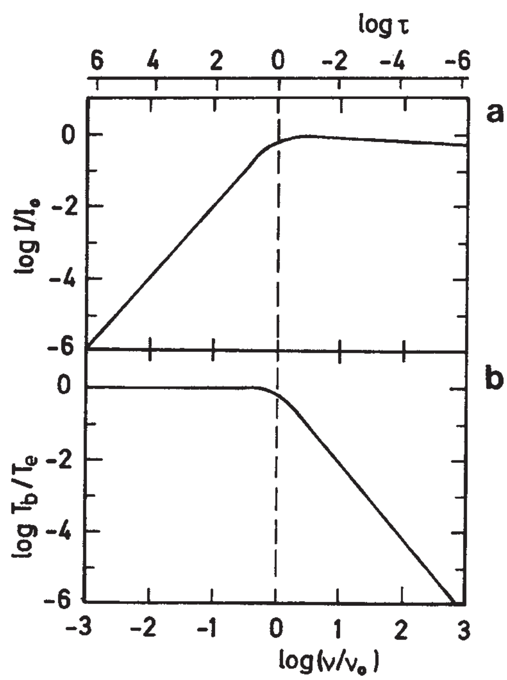

Fig. 1. *Thermal radiation of the ionized gas cloud. (a) Spectral distribution of the intensity. (b) Spectral distribution of the temperature with respect to the brightness (Rohlfs & Wilson 1996).*

Considering the model of the interstellar medium (M&O) created by the shock waves of the supernova explosions (see Fig. 2), we perform the analysis taking into account the transparency of the medium with respect to the radiation at 1 GHz using Eq. (5). For the interstellar medium defined by the previously stated parameters – that is, the so-called HIM ($T \approx 4.5 \times 10^5$K, $n \approx 3.5 \times 10^{-3}$cm^{-3}) – the frequency ν_0 is within the kilohertz range. For the warm neutral medium (WNM) ($T \approx 8000$K, $n \approx 0.37$cm^{-3}) ν_0 is in the vicinity of 3 MHz, and for the warm ionized medium (WIM) ($T \approx 8000$K, $n \approx 0.25$cm^{-3}), ν_0 is in the vicinity of 2 MHz. Cold cores of small interstellar clouds with temperatures $T = 80$K and concentrations $n = 42\,$cm^{-3} have a "break" in the spectrum at approximately 5 GHz. Our previous considerations lead us to conclude that SNRs are transparent to radiation at 1 GHz (except in the cases of cold cores of the small interstellar clouds). Therefore, we can detect radio radiation at

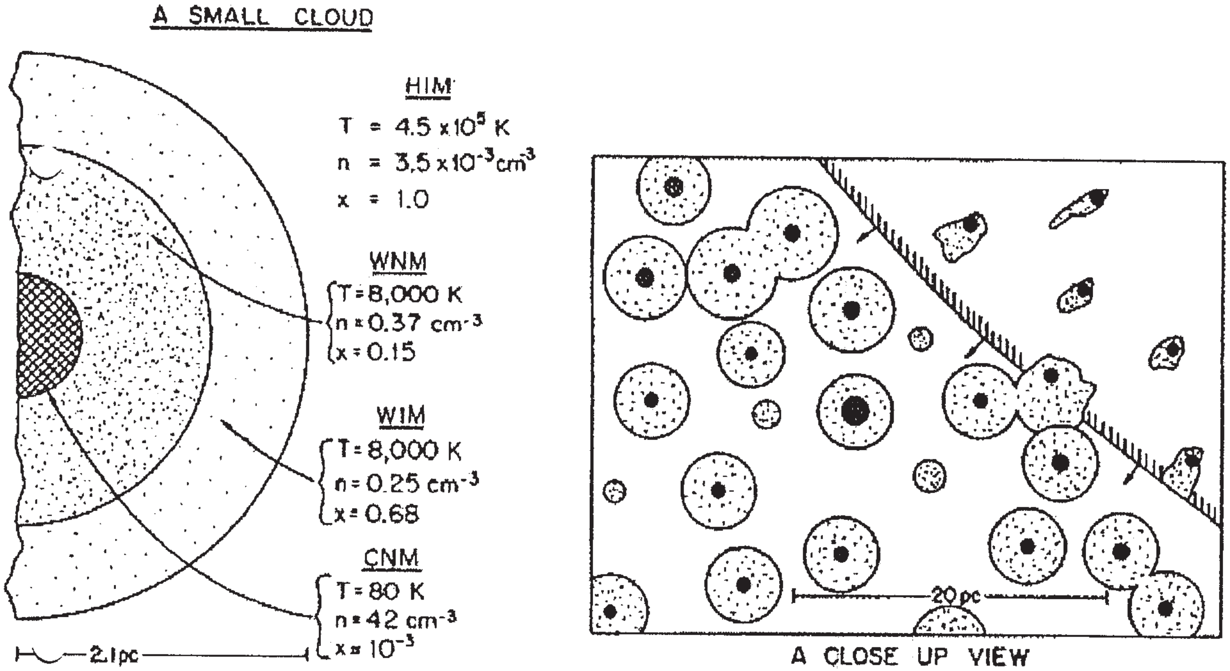

Fig. 2. *(Left) A segment of the average small interstellar cloud. The central part represents the cold core of the cloud (CNM). The middle layer is the warm neutral medium (WNM), and the outer layer represents the warm ionized medium (WIM). Concentrations and temperatures for each of these cases are given.*

(Right) A region of the interstellar matter of a size of 30×40 *pc. The shock wave of the supernova is expanding from the upper right corner of the picture. The cold core radii (black circles) are in the interval of 0.4 - 1 pc. All of the clouds with cores have warm wrapping (dotted areas) with radii around 2.1 pc. Some clouds are too small to have cold cores. Cloud envelopes inside the SNR are compressed and destroyed (from M&O).*

this frequency from the inside of the SNR, and as the SNR expands we can expect the amount of thermal radiation flux from the optically thin interior to expand as well.

According to the theory of M&O, interstellar clouds evaporate in the hot medium created by the shock wave, and only cold cores remain in the SNR interior. The structure of the interstellar cloud and a part of the interstellar medium that is swept up by the SNR shock wave are presented in Fig. 2.

This model of the interstellar matter leads to the conclusion that mainly cold cores can absorb radiation at the observed radio frequencies, and that the emplacement of the cores between the SNR and the observer can be a reason why SNRs can not be always seen completely.

2.2. $\Sigma - D$ relation for synchrotron radiation and thermal radiation from the ionized gas cloud (the case of constant temperature)

We have already mentioned the mathematical procedure which can describe the interaction between two functional dependencies – that is, the convolution method. The convolution of two functions f(x) and g(x) is the function h(t) which is defined by the following integral:

$$h(t) = \int_{-\infty}^{\infty} f(x) g(t - x) dx, \qquad (6)$$

or symbolically

$$h(t) = f(x) * g(x). \qquad (7)$$

The final result of the Shklovsky theory is the relation $\Sigma \propto D^{-6}$, for $\alpha = 0.5$ (average spectral index for SNRs), where flux density $F_\nu \propto \nu^{-\alpha}$. If relation (4) is convoluted with the previous relation we come up with an integral form:

$$\Sigma(t) \propto \int_{0}^{\infty} \frac{R}{(t - R)^6} dR. \qquad (8)$$

Here the integrals are evaluated over the range of $R = 0$ through $R = +\infty$ to describe the expansion of the SNR from very small values (nearly zero) at the beginning of the explosion to very large values (limiting case is ∞) at the end of its lifetime. This integral has the following solution:

$$\Sigma(t) \propto \frac{1}{20} t^{-4}. \qquad (9)$$

We therefore conclude that the relation derived from the combination of the results of the Shklovsky theory and the results acquired in Section 2.1 has the following form:

$$\Sigma_\nu \propto D^{-4}, \tag{10}$$

Therefore, the introduction of the thermal component into the Shklovsky relation leads to a form of the $\Sigma - D$ relation which is closer to the empirical result.

The theoretical model where the shell thickness remains constant (Kesteven 1968) yields a relation in the following form: $\Sigma \propto D^{-4.5}$ (for $\alpha = 0.5$). Assuming a shell model for the SNR, in accordance with the model of M&O (that is, an SNR with hot interior), thermal flux from hot interior can be expected. The convolution integral in this case is:

$$\Sigma(t) \propto \int_0^\infty \frac{R}{(t-R)^{4.5}} dR = \frac{4}{35} t^{-2.5}. \tag{11}$$

Similarly to the previous convolution example, this relation (11) becomes:

$$\Sigma_\nu \propto D^{-2.5}. \tag{12}$$

This relation has a value for β which is closest to the latest Galactic empirical $\Sigma - D$ relations obtained by C&B for the average values of the spectral index $\alpha = 0.5$.

3. DISCUSSION AND CONCLUSIONS

Models of Poveda & Woltjer (1968) and D&S convoluted with the $\Sigma - D$ relation model for the ionized gas cloud produce $\Sigma - D$ relations which do not agree with empirical relations. Relations derived in this manner have a very flat slope. For example, the convolution of relation (4) and the D&S relation $\Sigma \propto D^{-3.5}$ results in the dependence $\Sigma \propto D^{-1.5}$. Similarly, the relation of Poveda & Woltjer ($\Sigma \propto D^{-3}$) is derived in the form of $\Sigma \propto D^{-1}$. General modification of the original Shklovsky theory is described in the previous two unmodified models while the modified Kesteven model is in the best accordance with the empirical data. At this point, it is necessary to emphasize that the consistency between the empirical relation of C&B $\beta = 2.38$ and the relation of D&S ($\beta = 3.5$), which was used to support their empirical relation, is poorer than the consistency of the same relation and the relation for the modified model of D&S ($\beta = 1.5$). Therefore, the latest theoretical relation is of the approximately same level of inconsistency with the empirical one, while the initial relations (Shklovsky 1960a, Kesteven 1968) are significantly improved.

By modifying Shklovsky theory, we have obtained a relation which is in closer agreement with the empirical results. The modified theoretical relation of Kesteven (1968) gives the best agreement with the updated Galactic empirical $\Sigma - D$ relation (C&B).

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МОДИФИКОВАНА ТЕОРИЈСКА $\Sigma - D$ РЕЛАЦИЈА: I. СЛУЧАЈ КОНСТАНТНЕ ТЕМПЕРАТУРЕ УНУТАР ОСТАТКА СУПЕРНОВЕ

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Претходно саопштење

Модификација теоријске $\Sigma - D$ релације за остатке супернових звезда у адијабатској фази ширења заснована је на конволуцији првобитне релације, коју је извео Шкловски, са $\Sigma - D$ релацијом за закочно зрачење јонизованог гасног облака (изведена у овом чланку). Ми смо користили модел Мекија и Острајкера за остатке супернових и међузвездану средину. Модификована теорија Шкловског даје решење које је блиско емпиријским резултатима. Кестевенова модификована теоријска релација даје најбоље слагање са Галактичком емпиријском $\Sigma - D$ релацијом.