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Preliminary report

HOW MANY UNIVERSES ARE NECESSARY FOR AN ICE CREAM TO MELT?

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SUMMARY: We investigate a quantitative consequence of the Acausal-Anthropic approach to solving the long-standing puzzle of the thermodynamical arrow of time. Notably, the size of the required multiverse is estimated on the basis of the classical Boltzmann connection between entropy and probability, as well as the thermodynamic properties of black holes.

Key words. cosmology: theory – gravitation – history and philosophy of astronomy

1. INTRODUCTION: THE MYSTERY OF THE THERMODYNAMICAL ASYMMETRY

The puzzle of the origin of the thermodynamical temporal asymmetry ("the arrow of time") is the following. In our experience, systems increase in entropy in the forward direction of time. Water and ink mix when together in a bottle; ice cubes in a drink melt, keeping the temperature of the drink constant, and after they have all melted, the temperature of the fluid will reach that of the environment. The underlying dynamical laws which are taken to govern thermodynamic systems, however, are symmetric in time: statistical mechanics predicts that entropy is overwhelmingly likely to increase in both temporal directions. Thus, ice is *prima facie* as probable to melt in contact with warm liquor as it is to freeze further; the same applies to a chunk of ice cream on a hot day. But we see it **always** melting down, never freezing. So where does this asymmetry of thermodynamics (and of our experience generally) come from?

The question has been highlighted in Boltzmann's time by people like Loschmidt, Culverwell, Burbury and Zermelo. Boltzmann himself was keenly aware of the puzzle. His first attempt to solve it, in the form of his (unjustly) celebrated H-theorem has failed, and in his later work, and in particular in the polemical exchange with Zermelo in 1896/97 (cf. Steckline 1983), he clearly realized that another solution is necessary (Boltzmann 1964). He offered two basic ideas and, interestingly enough, both are essentially **cosmological**, i.e. pertaining to the properties of universe at large and its origin. First, he speculated that the universe is of finite age and, having started in a low-entropy state, has not simply had time to reach the equilibrium yet (the initial conditions hypothesis). Secondly, he–attributing the idea to his assisstant, Dr. Shuetz–suggested that our ordered part of the universe is a fluctuation within the much larger universe which is eternally existing and in equilibrium almost everywhere (the anthropic fluctuation hypothesis). Historically, the first of these ideas has been taken more seriously; but in recent years we are witnessing a significant revival of the entire topic of the origin of the second law and the thermodynamical asymmetry (e.g. Zeh 1992; Price 1996, 2002; Lieb & Yngvason 1999; Uffink 2001; Uffink & Brown 2001; Kutrovátz 2001).

Nowadays, in particular after the explosive development of quantum cosmology during the last two decades, it is possible to unify both Boltzmann's ideas into a unified framework, which can appropriately, and by analogy with other possibilities (Price 2002), be dubbed the Acausal-Anthropic explanation. The basic idea of the Acausal-Anthropic approach is the following: having already received from quantum cosmology a useful notion of the multiverse, we could as well employ it in order to account for the *prima facie* extremely improbable choice of (local) initial conditions. In other words, we imagine that everything that exists, for which we shall use the term multiverse, represents a "Grand Stage" for unfolding of–among other things–thermodynamical histories of chunks of matter. Entropy *in the multiverse* is almost everywhere high at all times ("almost" here means "everywhere minus possible subset of a very small or zero measure"). Our cosmological domain ("the universe") represents a natural fluctuation–presumably of very small or zero measure; but the anthropic selection effect answers the question why do we find ourselves on an upward slope of such a fluctuation. We have discussed basic properties of the new solution and its relationship to other solutions elsewhere (Ćirković 2002; Ćirković and Milošević-Zdjelar 2003). Here we wish to concentrate upon the particular issue of the **size** of the multiverse required for explanation of the observed low cosmological entropy. In this we follow the lead of Penrose (1979, 1989), who was the first to estimate the size of the (classical) phase space necessary to accomodate a low-entropy Big Bang.

2. THE NUMBER OF DEGREES OF FREEDOM AND SIZE OF THE MULTIVERSE

How many domains are required in order to account for the observed thermodynamical asymmetry? While the exact answer is difficult to conceive, we may go after Penrose and use the Bekenstein-Hawking formula (Bekenstein 1973; Hawking 1974) to estimate the **lower limit** on the size of ensemble of domains in which we expect to find one similar to ours on purely probabilistical grounds. According to this formula, the entropy of a black hole is

$$S_{BH} = \frac{c^3}{G\hbar}\frac{A}{4}, \tag{1}$$

where A is the area of the **horizon** of the black hole. Schwarzschild's solution then gives us

$$A = 4\pi\, R_{Sch}^2 = 4\pi \left(\frac{2Gm}{c^2}\right)^2 = \frac{16\pi\, G^2}{c^4} m^2. \tag{2}$$

This corresponds to the value of entropy

$$S_{BH} = \frac{4\pi\, G}{\hbar c} m^2, \tag{3}$$

Obviously, the celebrated Boltzmann's formula $S = \ln W$ (in "natural" $k = 1$ units we shall use in the entire discussion) suggests that the required number is $N_{\rm min} \sim \exp\left(S_{\rm max} - S_0\right)$, where $S_{\rm max}$ is the entropy of the state of maximal probability of the matter in our domain (what would be traditionally called the entropy of the "heat death" state), and S_0 is the "realistic entropy" of the same matter.[1] However, our domain is limited by our particle horizon at present, and will be almost certainly limited (cf. Krauss and Turner 1999) by an **event horizon**, due to the contribution Ω_Λ of the vacuum energy density ("cosmological constant"). Numerically, the difference between the two in the realistic case is not very large in cosmological terms (∼1 Gpc), so we will not make a big error in attributing the state of low entropy to those currently invisible (but visible to our descendants!) parts of our domain between the particle and the event horizon. Thus, we need to account for entropy of matter of cosmological density Ω_m (predominantly in CDM or similar particles, with ∼15-20% of baryons). Assuming that our domain is globally flat, with no net electric charge and no net angular momentum, from (3) we obtain:

$$\begin{aligned} N_{\rm min} &\sim \frac{\exp S_{\rm max}}{\exp S_0} = \frac{1}{\exp S_0} \exp\left(\frac{c^3}{G\hbar}\frac{A}{4}\right) \\ &= \frac{1}{\exp S_0} \exp\left(\frac{4\pi\, G}{\hbar c} m^2\right) \\ &= \frac{1}{\exp S_0} \exp\left(\frac{\pi}{G\hbar c} H_0^4 \Omega_m^2 R_h^6\right), \end{aligned} \tag{4}$$

where H_0 is the present-day Hubble constant, and R_h size of the horizon. S_0 is the present-day entropy of the actual universe within the same spatial volume.[2] Using the case of an event horizon which is

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[1] More precisely, we should take the entropy of our domain **at the Big Bang**. However, we run into big conceptual difficulties having to do with quantum cosmology there. On the other hand, the error introduced by considering the **present-day** entropy is, as we shall see, negligibly small. In other words, the potential of our universe for interesting (i.e. entropy-increasing) events is so huge, that everything which has already occured is a negligible subset of that potentiality.

[2] There are some fine details here we can only briefly discuss. The size of observable universe is still smaller than the event horizon for currently favored cosmological model by about 4 × 10^9 light-years. The difference is larger, of course, if the contribution of the cosmological constant is smaller. We have to assume validity of the cosmological principle of Milne and Eddington (i.e. homogeneity and isotropy) on these spatial scales–which is not verifiable at present–if the calculation is to be meaningful. In addition, it is obvious that any amount of entropy we observe today has two parts: the one "inherited" from the Big-Bang initial conditions, and another created during the thermodynamical history of the universe. In the "standard model" (e.g. Peebles 1993), the former is overwhelmingly larger, and everything that has occured after the recombination epoch is just a small correction. However, this view has not been completely unquestioned: from time to time, models with low initial entropy ("cold Big Bangs") appear, in addition to more radical unorthodoxies rejecting the very notion of standard initial conditions (e.g. Layzer & Hively 1973; Hoyle 1975; Rees 1978; Aguirre 2000).

fixed by magnitude of the cosmological constant only (e.g. Gibbons and Hawking 1977),

$$R_h = \frac{c}{H_0 \sqrt{\Omega_\Lambda}}, \tag{5}$$

we obtain the following remarkable expression (for the flat $\Omega_m + \Omega_\Lambda = 1$ universe):

$$\begin{aligned} N_{\min} &\sim \exp\left(\frac{\pi\, c^5}{G\hbar} \frac{\Omega_m^2}{\Omega_\Lambda^3} \frac{1}{H_0^2} - S_0 \right) \\ &= \exp\left[\frac{\pi\, c^5}{G\hbar} \frac{\Omega_m^2}{(1-\Omega_m)^3} \frac{1}{H_0^2} - S_0 \right]. \end{aligned} \tag{6}$$

We notice the appearance of all major constants of nature (including the "silent" Boltzmann constant, which is omitted since we are working in "natural" units!) in this formula, with the exception of the elementary charge, which is reasonable since we are dealing with the standard electrically neutral universe. In addition, the total entropy of our cosmological domain, S_0, appears and it represents, in a sense, the outcome of all and every process which has taken place since the beginning of time!

How big is the realistic entropy S_0? The conventional answer is simple: the entropy is by far dominated by the photons of the cosmic microwave background, whose specific entropy ("entropy-per-baryon") is a well-known dimensionless number (e.g. Barrow & Tipler 1986):

$$s_{CMB} = \left(\frac{n_\gamma}{n_B} \right)_0 \approx 10^8, \tag{7}$$

where n_γ and n_B are number densities of photons and baryons respectively. Taking the standard estimate that the total number of baryons within horizon is $\sim 10^{80-81}$, we may be certain that S_0 is not larger than 10^{90} (again in natural units).

Common numerical values of the cosmological parameters Ω_m (≈ 0.3) and H_0 (≈ 60 km s^{-1} Mpc^{-1}) inserted in (6) give us stupendous double exponential

$$N_{\min} \sim \exp\left(1.9 \times 10^{121}\right). \tag{8}$$

At least that many domains in the multiverse are needed to account for the observed asymmetry in this manner (Penrose 1989).[3] (This is easily generalized to the case of charged or rotating universe characterized by some other set of parameters, but their exact values make no difference when numbers of such magnitude are involved. See Fig. 1, where the insensitivity of this huge number on the much-disputed value of the dimensionless Hubble constant h is shown.) This is the price one must pay for embedding the atypical initial conditions into a wider manifold. Of course, the total number of domains may be infinite, in which case the conclusions of Ellis and Brundrit (1979) will apply, and any worry about the "specialty" of our initial conditions is immediately discarded. On the truly global scale–i.e. in the multiverse–there is no thermodynamical asymmetry, no arrow of time. Only through an anthropic **selection effect** do we perceive one in our own cosmological domain (cf. Bostrom 2002). In a sense, the ice cream melts because such state-of-affairs is necessary for life and intelligence (not to mention ice-cream makers!) to occur.

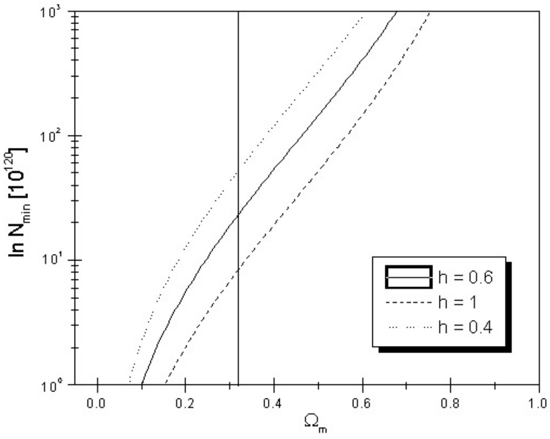

Fig. 1. *Minimal size of the multiverse as a function of cosmological density in matter for our domain. We notice that these values are largely insensitive to the observationally much-disputed value of Hubble constant (a realistic case h= 0.6 is shown with the two other extremes). Vertical line denotes what we today consider as the best measurement of* Ω_m.

3. DISCUSSION

We notice that the expression (6) is valid only for $\Omega_\Lambda \neq 0$. Formally, this is because in $\Omega_\Lambda = 0$ universes there is no event horizon, and any large-scale inhomogeneity will, sooner or later, enter the observer's particle horizon (e.g. Adams & Laughlin 1997). However, this points also to a deeper issue: we cannot have an empirical warrant that Boltzmann and Schuetz were not correct, after all. Since any large-scale inhomogeneity will represent a state of high (gravitational) entropy, hypothetical entering of such inhomogeneity into our view (particle horizon) at late cosmological epochs will mean that the entropy content of the accessible matter fields is sharply increased. The universe at **very large** scale may as well be very inhomogeneous, and that is tantamount to saying that the universe at those very large scales

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[3] The actual entropy S_0 is negligible, since $10^{121} - 10^{90} \approx 10^{121}$ with exactness higher than anything else in physics! That fact alone shows how exceptionally well-ordered the beginning and the entire history of our universe was in comparison to the generic case.

is of very high entropy–exactly the point of Boltzmann and Schuetz (compare Boltzmann 1895). On the other hand, the distinction between such a large universe and the multiverse we have postulated here seems to be semantical rather than physical. Only a cosmological constant can **ensure** that no such perturbations will **ever** enter our horizon in the future. Thus, the second law is not necessarily universal, but if we wish it to be forever valid within the domain of causally accessible matter fields, we need something like the cosmological constant Λ.

Now we see the way to quantify the famous Feynman's objection to the Boltzmann-Schuetz entropy-fluctuation picture: why the size of low-entropy fluctuation is so much larger than it was necessary for the emergence of intelligent observers on Earth (Feynman 1965)? It is possible to specify the probability of having low-entropy volume of the specified size in the manner sketched above. One could ask, finally, how do we know that the thermodynamical asymmetry is the same here and on some distant, but observable, galaxy? Does ice cream melt in NGC 5625 in the same manner as here? This, admittedly hardly empirical question is, in fact, the deepest epistemological issue of astronomy and related sciences. For the moment, we might state that this, like the homogeneity of the universe, represents a necessary metaphysical baggage of any usable astronomical theory. However, we might claim that the required device is something like the *postulate of uniform thermal histories* of Bonnor & Ellis (1986) and Collins (1990). The basic idea here is (Bonnor & Ellis 1986):

> that we *do* have an observational indication of one kind of homogeneity: namely, we believe we observe in distant regions astrophysical conditions essentially similar to those we observe nearby... Let us assume that the same laws of Nature apply throughout the Universe – which is, of course, the basis for our cosmological models. Similar astrophysical conditions observed in distant regions may then be taken to imply there have been *similar thermodynamic histories* of the matter in widely separated parts of the Universe–for if the thermodynamic histories had been significantly different, element formation would certainly have been different, and so presumably would star and galaxy formation.

This is almost trivially logical and brings, as Collins has shown, expected mathematical consequences.

Finally, it may be noticed that the closeness of the result above to 10^{120} reprepresents another instance of appearance of the ubiquitous Eddington-Dirac "large number" 10^{40}, which caused so much debates in physics and astronomy during the XX century (Dirac 1937, 1938; Dicke 1961; for a review see Barrow & Tipler 1986, chapter 4). Obviously, $N_{\rm min} \sim \left(10^{40}\right)^3$. Is this a real coincidence, or it has a Dicke-like explanation in terms of the age of the universe or other cosmological parameters? It seems that we cannot simply explain it in the same fashion, since it includes the Planck constant h (or η), which was absent from the classical-cosmological discourses of Dirac and Dicke. Although this issue deserves further investigation, it seems intuitively clear that a viable anthropic explanation could be found, contingent upon the spatial and temporal size of the universe (or a part of it) required for supporting intelligent life and observers. This is indicated by the "undercover" appearance of the age of the universe $t_0 \sim H_0^{-1}$ in (6).

In any case, we may again conclude that things of routine everyday occurence have indeed deep and crucial cosmological roots. Melting of ice cream, and indeed our very existence are important consequences of the architecture of reality on the largest scales. This just reiterates the significance and importance of cosmology not just within the scientific framework, but within the entire corpus of human thought and culture.

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REFERENCES

Adams, F.C. and Laughlin, G.: 1997, *Rev. Mod. Phys.* **69**, 337.

Aguirre, A.: 2000, *Astrophys. J.* **533**, 1.

Barrow, J.D. and Tipler, F.J.: 1986, The Anthropic Cosmological Principle (Oxford University Press, New York).

Bekenstein, J.D.: 1973, *Phys. Rev. D* **7**, 2333.

Boltzmann, L.: 1895, *Nature*, **51**. 413.

Boltzmann, L.: 1964, Lectures on gas theory, translated by S. G. Brush (University of California Press, Berkeley).

Bonnor, W.B. and Ellis, G.F.R.: 1986, *Mon. Not. Roy. Astron. Soc.* **218**, 605.

Bostrom, N.: 2002, Anthropic Bias: Observation Selection Effects (Routledge, New York).

Collins, C.B.: 1990, *Class. Quantum Grav.* **7**, 1983.

Ćirković, M.M.: 2003, *Found. Phys.*, **33**, 467.

Ćirković, M.M. and Milošević-Zdjelar, V.: 2003, *Foundations of Science*, in press.

Dicke, R.H.: 1961, *Nature*, **192**, 440.

Dirac, P.A.M.: 1937, *Nature*, **139**, 323.

Dirac, P.A.M.: 1938, *Proc. R. Soc.* **A338**, 439.

Ellis, G.F.R. and Brundrit, G.B.: 1979, *Q. Jl. R. Astr. Soc.* **20**, 37.

Feynman, R.P.: 1965, The Character of Physical Law (Cox and Wyman Ltd., London).
Gibbons, G.W. and Hawking, S.W.: 1977, *Phys. Rev. D* **15**, 2738.
Hawking, S.W.: 1974, *Nature*, **248**, 30.
Hoyle, F.: 1975, *Astrophys. J.* **196**, 661.
Krauss, L.M. and Turner, M.S.: 1999, *Gen. Rel. Grav.* **31**, 1453.
Kutrovátz, G.: 2001, *Open Systems and Information Dynamics*, **8**, 349.
Layzer, D. and Hively, R.: 1973, *Astrophys. J.* **179**, 361.
Lieb, E. and Yngvason, J.: 1999, *Phys. Rep.* **310**, 1.
Linde, A.D.: 1986, *Phys. Lett. B* **175**, 395.
Linde, A.D.: 1990, Inflation and Quantum Cosmology (Academic Press, San Diego).
Peebles, P.J.E.: 1993, Principles of Physical Cosmology (Princeton University Press, Princeton).
Penrose, R.: 1979, in General Relativity: An Einstein Centenary, ed. by Hawking, S. W. and Israel, W. (Cambridge University Press, Cambridge), p. 581.
Penrose, R.: 1989, The Emperor's New Mind (Oxford University Press, Oxford).
Price, H.: 1996, Time's Arrow and Archimedes' Point (Oxford University Press, Oxford).
Price, H.: 2002, *Brit. J. Phil. Sci.*, **53**, 83.
Rees, M.J.: 1978, *Nature*, **275**, 35.
Steckline, V.S. 1983, *Am. J. Phys.* 51, 894.
Uffink, J.: 2001, *Studies in History and Philosophy of Modern Physics* **32B**, 305.
Uffink, J. and Brown, H.: 2001, *Studies in History and Philosophy of Modern Physics* **32B**, 525.
Zeh, H.D.: 1992, The physical basis of the direction of time (Springer Verlag, New York).

КОЛИКО УНИВЕРЗУМА ЈЕ ПОТРЕБНО ДА БИ СЕ ОТОПИО СЛАДОЛЕД?

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Претходно саопштење

У овом раду истражујемо квантитативне последице Акаузално-антропичког приступа решавању старе загонетке порекла термодинамичке стреле времена. Величина траженог мултиверзума је процењена на основу класичне Болцманове релације ентропије и вероватноће, као и термодинамичких особина црних рупа.