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GUIDING OF WHISTLER-MODE WAVES WITH FREQUENCIES ABOVE ONE HALF OF THE GYROFREQUENCY IN THE MAGNETOSPHERE

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SUMMARY: Ray-tracing calculations show that a trough, $L = 4.6$, located in the region outside the plasmapause is capable to guide whistler-mode waves in frequency range $0.5f_{Bmin} - 0.64f_{Bmin}$, from one hemisphere to the other. From an initial position rays propagate in unducted mode up to $h \sim 14000$ km where they are trapped inside the trough. Propagation along the trough is possible only with great shifting of wave normal direction in respect to the direction of the geomagnetic induction $\mathbf{B}$, following the variations of the magnitude of the ambiental geomagnetic induction, or those of ratio f/f_B. On leaving the trough rays propagate in unducted mode downward in the magnetosphere.

1. INTRODUCTION

Many kinds of whistler-mode emissions in the VLF/ELF range are observed just around the plasmapause. This region is one of the places in the magnetosphere which is abundant in wave phenomena in different frequency ranges (Hattori *et al.*, 1991). Many authors have investigated various aspects of whistler- mode wave propagation near or inside plasmapause. They have analyzed waves in frequency range $f<0.5f_B$ (for example Inan and Bell, 1977; Lester and Smith, 1980; Sazhin and Varshavski, 1982; Hattori *et al.*, 1991).

High frequency $f>0.5f_B$ whistler-mode VLF emissions were discovered later. Hayakawa *et al.* (1984) have elucidated the detailed characteristics of the VLF emissons by determing the wave normal directions of the waves observed in the vicinity of the equator on the GEOS 2 satellite. High frequency whistler-mode VLF emissions are excited at the equator of the magnetosphere, at frequency above $0.5f_B$ and with wave normal close to the oblique resonance angle for whistler propagation (Muto and Hayakawa, 1987).

It is well known that whistlers propagating in the ducts (waveguide with an enhancement of electron density) have an upper cut-off frequency. This frequency is generally equal to one half of the minimum gyrofrequency along the ray path (Carpenter, 1968). Whistlers guided by or around inner side of the trough have an upper cut-off frequency $\sim 0.5f_{Bmin}$ (Šulić, 1991). Trapping of whistler mode waves with $f>0.5f_{Bmin}$ is possible only in troughs (Helliwell, 1965).

The main aim of this paper is an attempt to define the characteristics of the trough in the region outside of the plasmapause capable of guiding whistler-mode waves with frequencies $f>0.5f_B$. We restrict our analysis to cold plasma and symmetric winter-night, WN, model of the magnetosphere. Ray-tracing calculations have been done to define initial conditions which are favorable for rays to be trapped by a trough at high altitudes and propagate from hemisphere to hemisphere. On the basis of these calculations we define the region outside the plasmapause from dipole latitude $\alpha \sim -30^\circ$ to $+30^\circ$ where a trough could trap whistler-mode waves with $f > 0.5 f_B$.

2. THEORETICAL BASIS

The magnetosphere is the region of the terrestrial environment where the geomagnetic field exerts the dominating influence on the motions of charged particles. The medium consits of neutral particles and equal number of positive and negative charged particles. The cold plasma approximation allows writing the dispersion equation for various waves in a particularly simple form

$$N^2 = 1 - \frac{2X(1-X)}{2(1-X) - Y^2\sin^2\theta \pm Y\sqrt{Y^2\sin^4\theta + 4(1-X)^2\cos^2\theta}} \tag{1}$$

where:
$X = {f_p}^2/f^2$; f_p - electron plasma frequency; f - wave frequency
$Y = f_B/f$; f_B electron gyrofrequency
θ angle between wave normal **k** and the direction of **B**.

This is the well known Appleton-Hartree equation. It is clear from Eq. (1) that the refractive index is a function of the wave normal angle θ. Such a medium is called anisotropic. The angle wherewith $N \rightarrow \infty$ is called the resonance angle and is given by

$$\theta = \theta_r < \arccos\sqrt{\frac{X + Y^2 - 1}{XY^2}}. \tag{2}$$

Most of the known properties of the whistlers can be explained with the aid of the magneto-ionic theory (Budden, 1961; Ratcliffe, 1972). At whistler frequencies the wave is affected mainly by electrons, because of the relatively small mass and the effects of ions can often be neglected. In the magnetosphere for finite value of θ the folowing conditions

$$Y^2\sin^2\theta \gg 2 \mid 1 - X \mid$$
$$Y^4\sin^4\theta \gg 4(1-X)^2\cos^2\theta \tag{3}$$

are valid. These conditions are known as quasi-longitudinal approximation. After applying them in Eq. (1) the real part of refractive index is

$$\mu^2 = 1 + \frac{X}{Y\cos\theta - 1} \tag{4}$$

The relation (4) predicts wave propagation for $\mu^2 > 0$ and it takes effect when

$$\theta = \theta_r < \arccos Y^{-1}. \tag{5}$$

If $\theta \rightarrow \theta_r$ then $\mu^2 \rightarrow \infty$.

In an anisotropic medium the direction of energy flow, or ray direction, differs markedly from the direction of the wave normal. The wave phase velocity is given as $v_{ph} = c/\mu$. When the wave is not strongly damped or amplified and the dispersion of the medium is not larger than the wave packet, cluster of wave energy propagates with the group velocity. Restricting the analysis to whistler mode propagation in a cold dense plasma and neglecting the contribution of ions (Helliwell, 1965; Sazhin, 1993) the absolute value of $\mathbf{v_g}$ is

$$\mid \mathbf{v_g} \mid = \frac{cf_B}{f_p}\frac{\sqrt{Y\cos\theta - 1}}{Y^2\cos\theta} \cdot \cdot \sqrt{4(Y\cos\theta - 1)^2 - Y^2\sin^2\theta} \tag{6}$$

When $\theta \neq 0$ the direction of $\mathbf{v_g}$ does not coincide with the direction of wave normal **k**.

The corresponding angle between **B** and $\mathbf{v_g}$ direction can be determined from equation

$$\psi = \theta - arctan\frac{Y\sin\theta}{2(Y\cos\theta - 1)} \tag{7}$$

The angle θ changes during the wave propagation in the magnetosphere and causes the change of ψ. By Eq. (7) the direction of $\mathbf{v_g}$ with respect to **B** and **k** can be determinated. As whistler wave propagates in the magnetosphere, where f_p and f_B are vary changeable, directions of **k** and $\mathbf{v_g}$ with respect to **B** are also varied.

3. MAGNETOSPHERIC MODEL AND RAY-TRACING

The electron and ion densities are represented by field-aligned isothermal diffusive equilibrium model (Angermani and Thomas, 1964). The electron density at any point in the magnetosphere is calculated from the electron, ion (H^+, He^+ and O^+) and temperature profiles at a reference level of 900 km. The charged particle temperature and densities at the reference level are given by polynomial and exponential fits to satellite data for different seasonal and diurnal conditions (Denby *et al.,* 1980). The results presented in this paper are obtained by two-dimensional ray-tracing program (Strangeways, 1992), which is based on Haselgrove's equations. The refractive index (4) is calculated that the medium is treated as consisting of a cold collisionless plasma composed of electrons and three species of ions.

The geomagnetic field model used is a centered dipole with an electron gyrofrequency of 870 kHz on the ground at the equator.

It is assumed that whistlers in the magnetosphere propagate along waveguides i. e. field-aligned crests or troughs of electron density (Helliwell, 1965; Park and Helliwell, 1971). A trough with a Gaussian cross section can be incorporated into the model of the magnetosphere, which gives an additional depression in electron density. Namely,

$$N_e = N_{e0}(1 - \delta e^{-x^2/{\sigma_d}^2}) \tag{8}$$

where N_{e0} denotes the slowly varying background electron density; x is the separation (in km) between the field lines corresponding to the ray position and that of the trough center; δ is the electron density depression at the trough center and σ_d is the distance either side of the trough center where the electron density has a value of $N_{e0}(1-\delta/\sqrt{e})$. A depression of the electron density has the value of one- tenth of δ at $2.15\sigma_d$ either side of the trough center (Strangeways, 1991)

4. RESULTS

In this paper we examine how trough incorporated in the region outside the plasmapause can act to guide VLF waves with frequency above one half the gyrofrequency. The plasmapause is an important boundary in the inner magnetosphere where the equatorial electron density drops abruptly by a factor of 10 or 100. The plasmapause location varies with local time and geomagnetic condition, but the average location is around $L_{pp} \sim 4$ (Carpenter, 1966). Along the closed geomagnetic field line L is constant and $L = 1/cos^2\alpha$, where α is the dipole latitude. So in this paper we take $L_{pp} = 4.0$ defining the position of the inner edge of the plasmapause for WN model of the magnetosphere.

In our ray-tracing calculations a trough at $L = 4.6$ has been modelled with an electron density depression of 10% and effective width of 215 km in the equatorial plane. The full depression in a trough has been reached at 2100 km in the WN model of the magnetosphere. From that altitude the central depression at x = 0 has varied linearly from $\delta = -10\%$ to zero at 300 km altitude. Fig. 1 shows the meridional cross section of the magnetosphere, illustrating the position of a trough at $L = 4.6$.

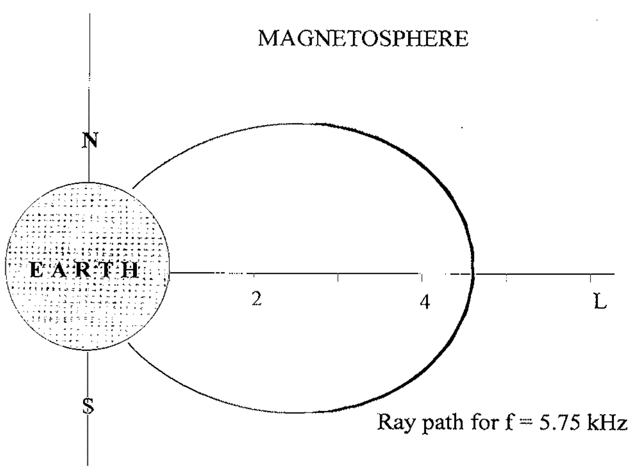

Fig. 1. *Meridional cross section of the magnetosphere. The light line illustrates the location of the trough. The heavy line shows the path of the trapped in the trough.*

In Šulić (1997) it has been shown that a whistler in a frequency range 1-4.4 kHz could be guided by the trough located at $L = 4.6$. The minimum electron gyrofreqency along the trough center at L = 4.6 is $f_{Bmin} = 9$ kHz. To analyze the effect of guiding VLF waves with frequencies greater than $0.5 f_{Bmin}$ by a trough, ray-tracing calculations have been done for rays in frequency range $f = 5 - 6$ kHz with 0.05 kHz steps. All rays started at 300 km altitude from an initial latitude of $\alpha = -60\overset{\circ}{.}95$. A wave normal angle to the vertical was changed from $-1°$ to $0°$ in so small a step as 10^{-3}. Rays in the frequency range $f = 5.3 - 5.8$ kHz successfully passed from one hemisphere to another. We have defined for these rays the initial wave normal angles, thus allowing them to reach 300 km altitude in the conjugate hemisphere with final wave normal angle inside the ionospheric transmission cone. Here are presented results only for 5.75 kHz ray, which started from initial position with wave normal angle to vertical of $-0\overset{\circ}{.}78$. Ray propagated in unducted mode from initial altitude up to altitude $h = 14300$ km and $\alpha = -32\overset{\circ}{.}50$ where it entered into the trough. In that position angle between **k** and **B** was $\theta = 53°$. Entering into the trough the ray propagated inside it (the heavy line in Fig. 1), crossed the eqatorial plane and left it in the opposite hemisphere at altitude h = 14500 km and $\alpha = 32\overset{\circ}{.}5$. Leaving the trough the ray propagated in unducted mode downward to the ionospheric levels. In the unducted mode both the wave normal and the ray direction can deviate significantly from the geomagnetic field direction. The attention of our work was to ray path characteristics inside the trough. For this part of the ray path inside the trough, conditions (3) are satisfied and quasi-longitudinal approximation can be used. The variation of ray path versus dipole latitude is shown in Fig. 2. The inner and outer sides of the trough are shown with dashed lines in Fig 2. As the ray moved in the trough, it crossed several times the trough center.

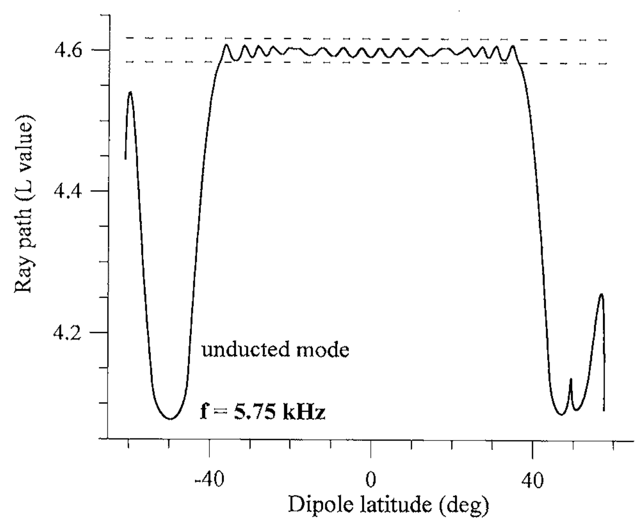

Fig. 2. *Ray path for f = 5.75 kHz versus dipole latitude.*

To test the obtained results we have done ray-tracing calculations for ray $f = 5.75$ kHz but starting from an altitude h = 14300 km and dipole latitde

$\alpha = -32\overset{\circ}{.}10$ with wave normal to $\mathbf{B}$ $\theta = 30°$. The same character of ray propagation is obtained. In the region of the magnetosphere from $\alpha = -32°$ to $+32°$ the trough $L = 4.6$ enables the propagation of the ray $f = 5.75$ kHz.

The ray exibits three different way of propagation through the trough. These differences are results of changeable electron gyrofrequency and directions of wave normal and group velocity in respect to $\mathbf{B}$.

The location of the trough is in the region outside the plasmapause, where is electron density is low. In the analyzed region electron plasma frequency changed from 19.2 kHz to 16.9 kHz, which means that $f_p > f$. The electron qyrofrequency falls off from 34 kHz to 9 kHz, as ray propagates from $\alpha = -32\overset{\circ}{.}10$ to the equatorial plane. In the conjugate hemisphere the electron gyrofreqency increases from 9 kHz to 34.5 kHz.

The effect of changing ratio f/f_B When the ray enters in the trough there is $f/f_B = 0.17$. As ray propagates through the trough to the equatorial plane the ratio f/f_B increases to 0.64. Around the equatorial plane the ray experiences the minimum gyrofrequency along the path. In the opposite hemisphere the behaviour of f/f_B is reverse, it decreases up to 0.17 at the position when the ray leaves the trough. In general there are two principal regions of interest, one with $0.1 < f/f_B < 0.5$ and other with $f/f_b > 0.5$. The gray segment in Fig. 3 represents region where $f/f_B > 0.5$. This ratio $f/f_B = Y^{-1}$ is the parameter for calculating the resonance angle (5). The resonance angle versus dipole latitude is shown in Fig. 3 (dashed lines).

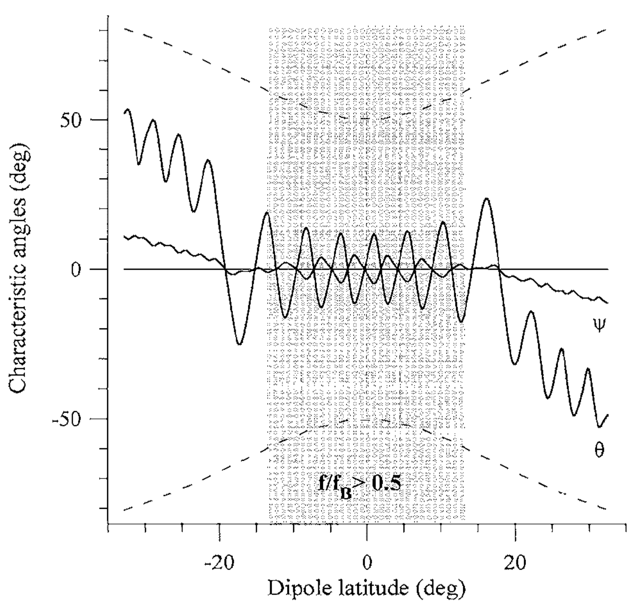

Fig. 3. *Characteristics of wave normal angle and angle between group velocity to* $\mathbf{B}$ *versus dipole latitude for ray $f = 5.75$ kHz.*

The behaviour of wave normal direction to $\mathbf{B}$ has been studied along the trapped ray in the trough $L = 4.6$. The angle θ versus dipole latitude is shown with heavy line in Fig. 3. In the segment of the trough, where $f/f_B < 0.5$, wave normal and group velocity are always on the same side of $\mathbf{B}$. The ray is trapped with $\theta = 53^0$. As ray approaches the segment where $f/f_B \sim 0.5$ θ slowly decraeses. There are significant differences between θ and resonance angle for the whole analyzed part of the ray path. Using Eq. (7) the angle betweeen $\mathbf{v_g}$ and $\mathbf{B}$ has been calculated and is shown by light line in Fig. 3. Along the whole analyzed ray path in the trough this angle has values $\psi < | 12° |$. In the segment where $f/f_B < 0.5$ the direction of $\mathbf{v_g}$ is on the same side as wave normal in respect to $\mathbf{B}$. The deviation of the direction of whistler-mode group velocity from the direction of $\mathbf{B}$ is less than the deviation of wave normal.

In the segment of the trough, where $f/f_B > 0.5$ the ray changes the mode of propagation. The group velocity moves to the left when the wave normal is on the rigth in respect to $\mathbf{B}$. The group velocity rotates in the opposite sense to that of the wave normal and therefore rotates away from regions of increasing refractive index. When both angles θ and ψ have values zero, ray path crosses the trough center and at that points k and $\mathbf{v_g}$ are parallel with $\mathbf{B}$.

In the opposite hemisphere, when the ray leaves region where $f/f_B > 0.5$ the wave normal and group velocity are again on the same side in respect to $\mathbf{B}$. As ray propagates through the trough where ratio f/f_B is decreasing the direction of wave normal moves away from the direction of $\mathbf{B}$.

5. CONCLUSION

We have carried out ray-tracing computation for the whistler-mode waves in frequency range $f = 5.3 - 5.8$ kHz, which coresponds to $f/f_B = 0.6 - 0.64$. The results show that these rays could be trapped at an altitude $h \sim 14000$ km in the trough $l = 4.6$. This propagation is possible with great displacement of wave normal direction in respect to $\mathbf{B}$, following the variations of the magnitude of the ambiental geomagnetic induction, or ratio f/f_B

(1) Starting from an initial altitude of 300 km all rays propagate in unducted mode up to $h \sim 14000$ km where they are trapped by the trough. Leaving the trough at similar altitude in the opposite hemisphere, they propagate as unducted mode downward.

(2) Rays are trapped in the trough in the region outside of the plasmapause from $\alpha \sim -32°$ to $+32°$.

(3) Inside the trough the rays change the mode of propagation due to the variation of the ratio of f/f_B.

(4) In the trough the trapped ray has θ values less than resonance angle. This difference is larges than 30°.

(5) The range of initial wave normal angle, from which starting rays could propagate interhemispherically and could be received at the ground, is very, very narrow.

REFERENCES

Angerami, J.J. and Thomas, J.O.: 1964 *J. Geophys. Res.,* **69** 4537.

Budden, K.G.: 1961, The Propagation of Radio Waves, Cambridge University Press.

Carpenter, D.L.: 1966, *J. Geophys. Res.*, **69**, 4537.

Carpenter, D.L.: 1968, *J. Geophys. Res.* **73**, 2919.

Denby, M. Bullough, K. Alexander, P.D. and Rycroft, M.J.: 1980, *J. Atmos. Terr. Phys.*, **42**, 51.

Hatori, K. Ishikawa, K. and Hayakawa M.: 1991, *Planet. Space Sci.,* **39**, 425.

Hayakawa, M. Parrot, M. and Lefeuvre, F.: 1986, *J. Geophys. Res.* **91**, 7989.

Helliwell, R. A.: 1965, Whistlers and Related Ionospheric Phenomena, Stanford University Press, Stanford.

Inan, U.S. and Bell, T.F.: 1977, *J. Geophys. Res.* **82**, 2819.

Lester, M. and Smith A.J.: 1980, Memories of National Institute of Polar Research, Special Issiue No.16, 113.

Muto, H. and Hayakawa M.: 1987, *Planet. Space Sci.*, **35**, 1397.

Park, C.G. and Helliwell, R.A.: 1971, *Radio Science*, **6**, 299.

Ratcliffe, J.A., 1972, An Introduction to the Ionosphere and Magnetosphere, Cambridge University Press.

Sazhin, S.S.: 1993, Whistler-mode waves in a hot plasma, Cambridge University Press.

Sazhin, S.S. and Varshavski, S.P.: 1982, *Geomagn. Aeronomy*, **20**, 719.

Strangeways, H.J.: 1991, *J. Atmos. Terr. Phys.* **53**, 151.

Strangeways, H.J.: 1992, CDCTRACE ray-tracing program (private communications).

Šulić, D.M.: 1997, *J. Atmos. Solar-Terr. Phys.* **59**, 569.

ПРОСТИРАЊЕ ТАЛАСА У МОДУ ЗВИЖДУКА СА ФРЕКВЕНЦИЈАМА ИЗНАД ЈЕДНЕ ПОЛАВИНЕ ОД ЖИРОФРЕКВЕНЦИЈЕ КРОЗ МАГНЕТОСФЕРУ

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Оригинални научни рад

Рачунања ray-tracing методом су показала да увала (таласовод са смањеном концентрацијом електрона у односу на околну плазму) смештена на L= 4.6 у делу магнетосфере иза плазмапаузе, може да проводи таласе у моду звиждука у фреквентном опсегу $0.6 f_B$ – $0.64 f_B$ од једне до друге хемисфере. Таласи са почетних позиција у неканалисаном моду простиру се нагоре до висине $h \sim 14000$ km, где улазе у увалу. Простирање унутар увале је могуће само са великим померањем правца вектора таласне нормале у односу на правац геомагнецке индукције **B**, пратећи промене интензитета геомагнецке индукције у окружењу или промену односа f/f_B. Напуштањем увале у супротној хемисфери талас се у неканалисаном моду простире надоле кроз магнетосферу.